

Advances in strength theories for materials under complex stress state in the 20th Century

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It is 100 years since the well-known Mohr-Coulomb strength theory was established in 1900. A considerable amount of theoretical and experimental research on strength theory of materials under complex stress state was done in the 20th Century. This review article presents a survey of the advances in strength theory (yield criteria, failure criterion, etc) of materials (including metallic materials, rock, soil, concrete, ice, iron, polymers, energetic material, etc) under complex stress, discusses the relationship among various criteria, and gives a method of choosing a reasonable failure criterion for applications in research and engineering. Three series of strength theories, the unified yield criterion, the unified strength theory, and others are summarized. This review article contains 1163 references regarding the strength theories. This review also includes a brief discussion of the computational implementation of the strength theories and multi-axial fatigue. [DOI: 10.1115/1.1472455]

1 INTRODUCTION

Strength theory deals with the yield and failure of materials under a complex stress state. Strength theory is a general term. It includes yield criteria and failure criteria, as well as multiaxial fatigue criteria, multiaxial creep conditions, and material models in computational mechanics and computer codes. It is an important foundation for research on the strength of materials and structures. Strength theory is widely used in physics, mechanics, material science, earth science, and engineering. It is of great significance in theoretical research and engineering application, and is also very important for the effective utilization of materials. Particularly for design purposes, it is important that a reliable strength prediction be available for various combinations of multiaxial stresses. It is an interdisciplinary field where the physicist, material scientist, earth scientist, and mechanical and civil engineers interact in a closed loop.

Strength theory is a very unusual and wonderful subject. The objective is very simple, but the problem is very complex. It is one of the earliest objectives considered by Leonardo da Vinci (1452-1519), Galileo Galilei (1564-1642), Coulomb (1736-1806), and Otto Mohr (1835-1918), but it is still an open subject. Considerable efforts have been devoted to the formulation of strength theories and to their correlation with test data, but no single model or criterion has emerged which is fully adequate. Hundreds of models or criteria have been proposed. It seems as if an old Chinese said: "Let a hundred flowers bloom and a hundred schools of thought contend."

Timoshenko (1878-1972) was an outstanding scientist, distinguished engineer, and a great and inspiring professor.

Timoshenko's summers of the years from 1903 to 1906 were spent in Germany where he studied under Foppl, Prandtl, and Klein. After his return from Germany in 1904, he wrote his first paper on the subject of "various strength theories" in 1904 [1]. "Strength theories" was also the title of sections in two of his books [2,3]. Now, "strength theories" is a chapter of most courses of "Mechanics of Materials," sometimes referred to as "Strength of Materials." Moreover, "yield criteria" or "failure criteria" is a chapter of most courses in Plasticity, Geomechanics, Soil Mechanics, Rock Mechanics, and Plasticity of Geomaterials, etc.

This subject, although there are some review articles and books, is difficult and heavy to survey. Some of the surveys were contributed by Mohr [4], Westergaard [5], Schleicher [6], Nadai [7,8], Marin [9], Gensamer [10], Meldahl [11], Dorn [12], and Prager [13] in the first half of the 20th century. It was also reviewed by Freudental and Geiringer [14], Naghdi [15], Filonenko-Boroditch [16], Marin [17], Paul [18], Goldenblat and Kopnov [19], and Taira (creep under multiaxial stress) [20] in the 1960s. This subject was further reviewed by Tsai and EM Wu (anisotropic material) [21], Bell (experiments) [22], Krempl [23], EM Wu (anisotropic failure criteria) [24], Michino and Findley (metals) [25], Salencon (soil) [26], Geniev *et al* (concrete) [27] in the 1970s; and by Yu [28,29], Zyczkowski [30], WF Chen (concrete) [31], Ward (polymer) [32], WF Chen and Baladi (soils) [33], Hamza (ice) [34], Shaw [35], Hosford [36], Rowlands [37], Ikegami (low temperature) [38], and Desai [39] in the 1980s. Strength theories were subsequently reviewed by Klausner [40], WF Chen [41,42], Du [43], Jiang (concrete) [44], Andreev (rock) [45], Shen (rock, soil) [46], Kerr (ice) [47], Gao and Brown (Multiaxial fatigue) [48], You and SB Lee (Mul-

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tiaxial fatigue) [49], Sheorey (rock) [50], WF Chen (concrete) [51], Yu, Zhao, and Guan (rock, concrete) [52], Shen and Yu [53], and Munx and Fett (ceramics) [54] in the 1990s. Two books regarding strength theory are published [55,56].

The advances in strength theories of materials under complex stress state in the 20th century will be summarized in the framework of continuum and engineering application.

2 STRENGTH THEORIES BEFORE THE 20th CENTURY

2.1 Early work

Leonardo da Vinci (1452-1519) and Galileo Galilei (1564-1642) were among the most outstanding scientists of that period. They may be the earliest researchers of the strength of materials and structures. Da Vinci and Galileo did tensile tests of wire and stone, as well as bending tests. Da Vinci believed that the strength of an iron wire would depend significantly on its length. Galilei believed that fracture occurs when a critical stress was reached [2].

Coulomb (1736-1806) may be the first researcher in the maximum shear stress strength theory. No other scientist of the eighteenth century contributed as much as Coulomb did to the science of mechanics of elastic bodies [2]. Coulomb's Memoir Essay [57] was read by him to the Academy of France on March 10 and April 2, 1773, and published in Paris in 1776. The paper began with a discussion of experiments which Coulomb made for the purpose of establishing the strength of some kind of sandstone; then, Coulomb gave a theoretical discussion of the bending of beams, the compression of a prism, and the stability of retaining walls and arches.

Coulomb assumed that fracture is due to sliding along a certain plane, and that it occurs when the component of force along this plane becomes larger than the cohesive resistance in shear along the same plane. To bring the theory into better agreement with experimental results, Coulomb proposed that, not only should cohesive resistance along the shear plane be considered, but also friction caused by the normal force acting on the same plane. This was the first description of the famous Mohr-Coulomb Strength Theory.

2.2 Strength theories in the 19th century

There were three strength theories in the 19th century.

The maximum stress theory was the first theory relating to the strength of materials under complex stress. It considers the maximum or minimum principal stress as the criterion for strength. This criterion was assumed by such scientists as Lamé (1795-1870) and Rankine (1820-1872), and was extended with the well known textbook of Rankine's, *Manual of Applied Mechanics* [58], the first edition of which appeared in 1858 at Glasgow University, and was published in 1861, the 21st edition entitled *Applied Mechanics* being published in 1921. Only one principal stress σ_1 of the 3D stresses $\sigma_1, \sigma_2, \sigma_3$ was taken into account.

The second strength theory was the maximum strain theory. Mariotte (1620-1684) made the first statement on the maximum elongation criterion or maximum strain criterion [59]. Sometimes, it was called Saint-Venant's criterion or the

second Strength Theory in the Russian and Chinese literature, and the maximum normal stress criterion was called the First Strength Theory.

Maximum strain theory was generally accepted, principally under the influence of such authorities as the two French academicians Poncelet (1788-1867) and Saint-Venant (1797-1886) [60]. In this theory it is assumed that a material begins to fail when the maximum strain equals the yield point strain in simple tension. This theory does not agree well with most experiments. It was very popular at one time, but no one uses it today.

In 1864, Tresca presented two notes dealing with the flow of metals under great pressure to the French Academy [61]. He assumed that the maximum shear stress at flow is equal to a specific constant. It is called the Tresca yield criterion now. A maximum shear stress criterion was also proposed by Guest [62]. This theory gives better agreement with experiment for some ductile materials and is simple to apply. This theory takes two principal stresses σ_1 and σ_3 of the 3D stresses $\sigma_1, \sigma_2, \sigma_3$ into account, and the intermediate principal stress σ_2 is not taken into account.

Beltrami suggested that, in determining the critical values of combined stresses, the amount of strain energy should be adopted as the criterion of failure [63]. This theory does not agree with experiments and has not been used in plasticity and engineering. In 1856, Maxwell suggested that the total strain energy per unit volume can be resolved into two parts: 1) the strain energy of uniform tension or compression and 2) the strain energy of distortion. Maxwell made the statement in his letter to William Thomson as follows: "I have strong reasons for believing that when (the strain energy of distortion) reaches a certain limit then the element will begin to give way." Further on he stated: "This is the first time that I have put pen to paper on this subject. I have never seen any investigation of this question. Given the mechanical strain in three directions on an element, when will it give way?" [2]. At that time, Maxwell already had the theory of yielding which we now call the maximum distortion energy theory. But he never came back again to this question, and his ideas became known only after publication of Maxwell's letter in the 1930s. It took researchers considerable time before they finally developed [2] the theory identical with that of Maxwell.

Strength theory was studied by Foppl [64], Voigt [65], Mohr, Guest, and others at the end of the 19th century. Comparisons of strength theories as applied to various design problems were given in a paper by Marin [9] and a book by Nadai [66]. A comprehensive bibliography on strength theory before the 1930s can be found in the article by Fromm [67]. A discussion of various strength theories with a complete bibliography of the subject was given by Ros and Eichinger [68].

3 THREE SERIES OF STRENGTH THEORIES

Mohr used the stress circle method [69] in developing his theory of strength in 1900 [70]. Otto Mohr (1835-1918) was a very good professor. When thirty-two years old, he was already a well-known engineer and was invited by the Stut-

Stuttgart Polytechnicum Institute (Stuttgart University) to become the professor of engineering mechanics. His lectures aroused great interest in his students, some of whom were themselves outstanding, such as Bach and Foppl.

Foppl stated that all the students agreed that Mohr was their finest teacher [2]. Mohr always tried to bring something fresh and interesting to the students' attention. The reason for his students' interest in his lectures stemmed from the fact that he not only knew the subject thoroughly, but also had himself done much in the creation of the science which he presented.

Mohr made a more complete study of the strength of materials. He considered failure in broad senses; that is, it can be yielding of the material or fracture. Mohr's criterion may be considered as a generalized version of the Tresca criterion [61]. Both criteria were based on the assumption that the maximum shear stress is the only decisive measure of impending failure. However, while the Tresca criterion assumed that the critical value of the shear stress is a constant, Mohr's failure criterion considered the limiting shear stress in a plane to be a function of the normal stress in the same section at an element point.

Mohr considered only the largest stress circle. He called it the principal circle and suggested that such circles should be constructed when experimenting for each stress condition in which failure occurs. The strength of materials under a complex stress state can be determined by the corresponding limiting principal circle.

At that time, most engineers working in stress analysis followed Saint-Venant and used the maximum strain theory as their criterion of failure. A number of tests were made with combined stresses with a view to checking Mohr's theory [65,71,72]. All these tests were made with brittle materials and the results obtained were not in agreement with Mohr's theory. Voigt came to the conclusion that the question of strength is too complicated, and that it is impossible to devise a single theory for successful application to all kinds of structural materials [2].

The idea of Mohr's failure criterion may be tracked back to Coulomb (1773) [67]. This criterion is now referred to as the Mohr-Coulomb strength theory (failure criterion). In the special case of metallic materials with the same strength in tension and in compression, the Mohr-Coulomb strength theory is reduced to the maximum-shear stress criterion of Tresca [61].

When Otto Mohr was teaching at the Stuttgart Polytechnicum, his lectures caused August Foppl to devote most of his energy to study of the theory of structures. Like Mohr, Foppl's activity in both research and teaching at the Polytechnical Institute of Munich was remarkably successful. He was an outstanding professor and knew how to hold students' interest, although his classes were very large.

At that time, Foppl followed Saint-Venant's notion and used the maximum strain theory in deriving formulas for calculating safe dimensions of structures. But at the same time he was interested in the various other strength theories, and to clarify the question of which should be used, he conducted some interesting experiments. By using a thick-

walled cylinder of high-grade steel, he succeeded in making compressive tests of various materials under great hydrostatic pressures. He found that an isotropic material could withstand very high pressure in that condition. He designed and constructed a special device for producing compression of a cubic specimen in two perpendicular directions and made a series of tests of this kind with cement specimens. It is the earliest high-pressure test.

Haigh [73] and Westgaard [5] introduced the limit surface in a 3D principal stress space. The advantage of such space lies in its simplicity and visual presentation. It is called the Haigh-Wesagaard space or stress space. Photographs of geometric models of such surfaces corresponding to various yield criteria before 1944 can be found in the papers by Burzynski [74] and Meldahl [11]. The yield surface in the stress space can be transformed into the strain space [75–77]. A comparison of various strength theories applied in machine design before the 1930s was given by Marin [78].

Prandtl was a great scientist. He was in the Academics of the USA, France, and other countries. Strength theory was one of the subjects studied by him. Prandtl himself drew up his theory of the two types of fracture of solids and devised his model for slip [2]. At that time, Prandtl's graduate students worked mainly on strength of materials until Prandtl went to the USA in 1941. Von Karman did experimental work on the strength of stone under confining lateral pressure at Goettingen University, and did much work in aerodynamics in the USA; Nadai did important work on strength and plasticity at the Westinghouse Research Lab; Prager was leading the new research in strength and plasticity at Brown University, and several well-known professors worked at Brown University with Prager [79,80]; Flügge and Timoshenko worked at Stanford University.

A lot of strength theories and expressions were presented after Mohr. The proposed criteria and material models in the 20th century are too many, and it is difficult to classify them. Fortunately, a fundamental postulate concerning the yield surfaces was introduced by Drucker [81,82] and Bishop and Hill [83] with the convexity of yield surface determined. After that the convexity of yield surface was generalized to the strain space by Il'yushin in 1961. Since then the study of strength theory has been developing on a more reliable theoretical basis.

The convex region and its two bounds are most interesting. One method we used for representing these theories is to use the principal shear stresses τ_{13} , τ_{12} , τ_{23} and the normal stress σ_{13} , σ_{12} , σ_{23} acting on the same planes. Strength theories may be divided into three kinds (Fig. 1 and Table 1). Three principal shear stresses and relating normal stresses are

$$\tau_{ij} = \frac{1}{2}(\sigma_i - \sigma_j); \quad \sigma_{ij} = \frac{1}{2}(\sigma_i + \sigma_j); \quad i, j = 1, 2, 3$$

3.1 Single-shear strength theory (SSS theory)

This series of strength theories considers the maximum shear stress τ_{13} and the influence of the normal stress σ_{13} acting on the same section. It can be written mathematically as

$$F(\tau_{13}, \sigma_{13}) = C \quad (1)$$

Table 1. Summary of three series of strength theories

	SSS series (Single shear stress series of strength theory)	OSS series (Octahedral shear stress series of strength theory)	TSS series (Twin shear stress series of strength theories)
Model of element	Hexahedron	Isoclinal octahedron	Dodecahedron or orthogonal octahedron
Shear stress yield criterion	SSS yield criterion $\tau_{13}=C$, Tresca, 1864; Guest, 1900	OSS yield criterion $\tau_8=C$, von Mises 1913; Eichinger 1926; Nadai 1937	TSS yield criterion $\tau_{13}+\tau_{12}(\text{or } \tau_{23})=C$, Hill, 1950; Yu, 1961; Haythornthwaite, 1961
Shear strain yield criterion	SSS (Strain) yield criterion $\gamma_{13}=C$	OSS (Strain) yield criterion $\gamma_8=C$	TSS (Strain) yield criterion. $\gamma_{13}+\gamma_{12}(\text{or } \gamma_{23})=C$
Failure criterion	SSS failure criterion $\tau_{13}+\beta\sigma_{13}=C$	OSS failure criterion $\tau_8+\beta\sigma_8=C$	TSS failure criterion $\tau_{13}+\tau_{12}(\text{or } \tau_{23})+\beta\sigma_{13}$ $+\beta\sigma_{12}(\text{or } \sigma_{23})=C$
Slip condition	Coulomb, 1773 Mohr, 1882-1900 SSS Slip condition Schmid, 1924	Burzynski, 1928 Drucker-Prager, 1952 OSS Slip condition von Mises, 1926	Yu-He-Song, 1985 TSS Slip condition Yu-He, 1983
Cap model	SSS Cap model Roscoe 1963; Wei, 1964	OSS Cap model Baladi, Roscoe, Mroz	TSS Cap model Yu-Li, 1986
Smooth ridge model	SSS ridge model Argyris-Gudehus, 1973;	OSS ridge model Lade-Duncan, 1975 Matsuoka-Nakai, 1974	TSS ridge model Yu-Liu, 1988
Multi-parameter criterion	Ashton <i>et al</i> (1965) Hobbs (1964) Murrell (1965) Franklin (1971) Hoek-Brown (1980) Pramono-Willam (1989)	Bresler-Pister (1958); Willam-Warnke (1974); Ottosen (1977); Hsieh-Ting-Chen (1979) Podgorski (1985); Desai, de Boer (1988); Song-Zhao (1994); Ehlers (1995)	TSS Multi-parameter criterion Yu-Liu, 1988 (1990)

According to the shear stress, it may be referred to as the single-shear strength theory (SSS theory).

3.1.1 Single-shear yield criterion [61]

The expression is

$$f = \tau_{13} = C, \text{ or } f = \sigma_1 - \sigma_3 = \sigma_s \quad (2)$$

It is the one-parameter criterion of the SSS (single-shear strength) theory. This yield criterion is also referred to as the maximum shear stress criterion or the third strength theory in Russian and in Chinese. It is adopted only for one kind of material with the same yield stress both in tension and in compression $\sigma_t = \sigma_c = \sigma_s$.

The generalization of the Tresca criterion by adding a hydrostatic stress term σ_m was given by Sandel [30], Davi-

genkov [84], Drucker [85], Volkov [86], and Hara [87,88] and others. The limit surface is a pyramid with regular hexagonal cross section similar to the Tresca's. The expression of the extended Tresca criterion is

$$f = \tau_{13} + \beta\sigma_m = C \quad (3)$$

3.1.2 Single-shear strength theory (Mohr-Coulomb 1900)

The expression is

$$F = \tau_{13} + \beta\sigma_{13} = C, \text{ or} \\ F = \sigma_1 - \alpha\sigma_3 = \sigma_t, \quad \alpha = \sigma_t / \sigma_c \quad (4)$$

It is a two-parameter criterion of the SSS (single-shear strength) theory. It is the famous Mohr-Coulomb theory and is also the most widely used strength theory in engineering. The failure locus of SSS theory on the π plane (deviatoric plane) has the inner hexagonal threefold symmetry (lower bound) as shown in Fig. 1. It is interesting that Shield [89] was the first to publish the correct form of the Mohr-Coulomb limit locus in the deviatoric plane in 1955 [18]. It is also indicated by Shield that after the paper was completed, he learned that the correct yield surface was obtained previously by Professor Prager and Dr Bishop in an unpublished work [89]. Before Shield, the limit surface of the Mohr-Coulomb strength theory was always consistent with a sixfold symmetry hexagonal pyramid failure surface that is intercepted by a Tresca-type hexagonal cylinder.

Single-shear strength theory (Mohr-Coulomb 1900) forms the lower (inner) bound for all the possible convex failure surfaces coincided with the Drucker postulation on the de-

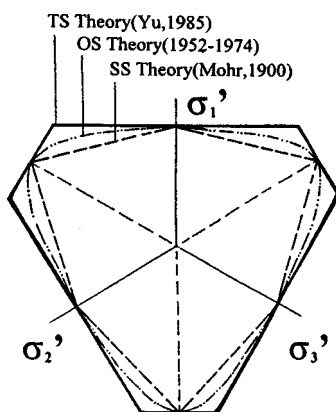


Fig. 1 Limiting loci of SSS, OSS, and TSS theories

viatoric plane in stress space. No admissible limit surface may exceed the Mohr-Coulomb limit surface from below, as shown in Fig. 1.

The disadvantage of the Mohr-Coulomb theory is that the intermediate principal stress σ_2 is not taken into account. Substantial departures from the prediction of the Mohr-Coulomb theory were observed by many researchers, *eg*, Shibata and Karube [91], Mogi [92,93], Ko and Scott [94], Green and Bishop [95], Vaid and Campanella [96], Lade and Musante [97], Michelis [98,99] and others.

3.1.3 Multi-parameter Single-Shear criteria

Multi-parameter Single-Shear criteria are nonlinear Mohr-Coulomb criteria (Mogi [92], Salencon [26], Hoek-Brown [100], *et al*) used in rock mechanics and rock engineering.

Some forms are expressed as follows

$$F = \tau_{13} + \lambda \sigma_{13}^n = 0 \quad \text{Murrell [101]} \quad (5)$$

$$F = \left(\frac{\sigma_1 - \sigma_3}{2c} \right)^n = 1 - \frac{\sigma_1 + \sigma_3}{2t} \quad \text{Ashton et al [102]} \quad (6)$$

$$F = (\sigma_1 - \sigma_3) + \sqrt{m\sigma_1 - c} = 0 \quad \text{Hoek-Brown [100]} \quad (7)$$

$$F = \left[(1-k) \frac{\sigma_1^2}{\sigma_c^2} + \frac{\sigma_1 - \sigma_3}{\sigma_c} \right]^2 + k^2 m \frac{\sigma_1}{\sigma_c} = k^2 c \quad \text{Pramono-Willam [103,104]} \quad (8)$$

in which $k \in (0,1)$ is the normalized strength parameter, c and m are the cohesive and frictional parameters.

3.1.4 Single-shear cap model

It is used in soil mechanics and engineering. It will be discussed in the next section.

3.1.5 Application of the SSS theory

Single-shear yield criterion (Tresca yield criterion) has been widely used for metallic materials and in mechanical engineering.

Mohr's theory (Single-shear strength theory) attracted great attention from engineers and physicists. "The Mohr-Coulomb failure criterion is currently the most widely used in soil mechanics" (Bishop [105]). "The Mohr-Coulomb theory is currently the most widely used for soil in practical applications owing to its extreme simplicity" [41,106]. "In soil mechanics, the Coulomb criterion is widely used; and in applied mechanics, Mohr's criterion has been widely used; for concrete Mohr-Coulomb criterion appears to be most popular." "Taking into account its extreme simplicity, the Mohr-Coulomb criterion with tension cutoffs is in many cases a fair first approximation and therefore suitable for manual calculation. However, the failure mechanism associated with this model is not verified in general by the test results, and the influence of the intermediate principal stress is not taken into account," as indicated by Chen [31].

SSS theory is the earliest and simplest series of strength theory. A considerable amount of research was done in connection with it. However, it is still studied up to the present (Shield [89]; Paul [18,107]; Harkness [108]; Pankaj-Moin

[109,110]; Heyman [111]; Schajer [112]). Multi-parameter Single-Shear criteria were used in rock mechanics and rock engineering.

3.2 Octahedral-shear strength theory (OSS Theory)

This series of strength theories considers the octahedral shear stress τ_8 and the influence of the octahedral normal stress σ_8 acting upon the same section. It can be written mathematically as

$$F(\tau_8, \sigma_8) = C \quad \text{or} \quad \tau_8 = f(\sigma_8) \quad (9)$$

This is a fruitful series in the strength theory. It contains:

3.2.1 Octahedral-shear stress yield criterion (von Mises yield criterion)

It is a one-parameter criterion of the OSS theory

$$f = \tau_8 = C, \quad \text{or} \quad J_2 = C, \quad \text{or} \quad \tau_m = C \quad (10)$$

It is the widely used yield criterion for metallic materials with the same yield stress both in tension and in compression. It is also referred to as the von Mises criterion [113], or octahedral shear stress τ_8 yield criterion by Ros and Eichinger [68] as well as Nadai [7,8]. Sometimes, it was referred to as the J_2 theory (second invariant of deviatoric stress tensor), shear strain energy theory (energy of distortion, Maxwell [2], Huber [114], Hencky [115]), equivalent stress criterion (effect stress or equivalent stress σ_e), or mean root square shear stress theory. It was also referred as the mean square shear stress τ_m averaged over all planes by Novozhilov [116], mean square of principal stress deviations by Paul [18], tri-shear yield criterion by Shen [46], and the fourth strength theory in Russian, Chinese, etc. All the expressions mentioned above are the same, because of

$$\begin{aligned} \tau_8 &= \frac{\sqrt{15}}{3} \tau_m = \frac{\sqrt{2}}{3} \sigma_e = \sqrt{\frac{2}{3} J_2} = \frac{2}{3} \sqrt{\tau_{12}^2 + \tau_{23}^2 + \tau_{13}^2} \\ &= \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \end{aligned} \quad (11)$$

3.2.2 Octahedral-shear failure criterion (Drucker-Prager criterion etc)

It is the two-parameter criterion of the OSS (Octahedral-shear strength) theory as follows

$$F = \tau_8 + \beta \sigma_8 = C. \quad (12)$$

This criterion is an extension of the von Mises criterion for pressure-dependent materials, and called the Drucker-Prager criterion expressed by Drucker and Prager [117] as a modification of the von Mises yield criterion by adding a hydrostatic stress term σ_m (or σ_8). The Drucker-Prager criterion was used widely in soil mechanics. The extended von Mises criterion, however, gives a very poor approximation to the real failure conditions for rock, soil, and concrete. It was indicated by Humpheson-Naylor [118], Zienkiewicz-Pande [119], and WF Chen [31,41,42] *et al*.

3.2.3 Multi-parameter octahedral shear failure criterion

The first effective formulation of such a condition in general form was given by Burzynski [74]. The general function of a three-parameter criterion is expressed as follows

$$F = A\tau_8^2 + B\sigma_8^2 + C\sigma_8 - 1 = 0 \quad \text{or}$$

$$F = \tau_8 + b\sigma_8 + a\sigma_8^2 = C. \quad (13)$$

The general equation (13) and its alternations, or particular cases, were later proposed, more or less independently, by many authors. The formulations of more than 30 papers were similar as indicated by Zyczkowski [30].

OSS theory contains many smooth (ridge) models and three, four, and five-parameter failure criteria used in concrete mechanics. Many empirical formulas, typically fitted with different functions, were proposed around the 1980s to cater to the various engineering materials. Among those were the ridge models and many multi-parametric criteria as follows:

$$F = \tau_8 + g(\theta) \left(C + \frac{1}{\sqrt{3}} \frac{\sigma_8}{\tau_8} \right) = 0 \quad \text{William et al [120]} \quad (14)$$

$$F = \frac{I_1 I_2}{I_3} = C \quad \text{Matsuoka-Nakai [121]} \quad (15)$$

$$F = \frac{I_1^3}{I_3} = C \quad \text{Lade-Duncan [122]} \quad (16)$$

$$F = \frac{3}{2} \tau_8^2 + \frac{1}{3} A \sigma_8 = C \quad \text{Chen-Chen [31]} \quad (17)$$

$$F = \frac{3}{2} \tau_8^2 - \frac{1}{6} \sigma_8^2 + \frac{1}{3} A \sigma_8 = C \quad \text{Chen-Chen [31]} \quad (18)$$

$$F = \tau_{8t} + a_1 \sigma_8 + a_2 \sigma_8^2 = C_1 \quad (\theta = 60^\circ)$$

$$F' = \tau_{8c} + b_1 \sigma_8 + b_2 \sigma_8^2 = C_2 \quad (\theta = 0^\circ) \quad \text{Willam-Warnke [120]} \quad (19)$$

$$F = \tau_8 + a\tau_8^2 + b\sigma_8 = C \quad \text{Ottosen [123]} \quad (20)$$

$$F = \left(\frac{I_1^3}{I_3} - 27 \right) \left(\frac{I_1}{p_a} \right)^m = C \quad \text{Lade [124]} \quad (21)$$

$$F = \tau_8 + a^2 \tau_8^2 + b\tau_8 + d\sigma_1 = C \quad \text{Hsieh et al [31]} \quad (22)$$

$$F = \tau_8 + a(\sigma_8 + b)^n = C \quad \text{Kotsovos [125]} \quad (23)$$

$$F = \alpha \sigma_m^2 + \beta \sigma_m + \gamma + (\tau_8 / g(\theta))^2 = 0 \quad \text{Zienkiewicz-Pande [119]} \quad (24)$$

where $g(\theta)$ is a shape function, and various functions were proposed as follows:

$$g(\theta) = \frac{2k}{(1+K) - (1-K)\sin 3\theta} \quad \text{Argyris-Gudehus [127]} \quad (25)$$

This function was improved by Lin-Bazant in [128] and Shi-Yang in [129] as follows:

$$g(\theta) = r_c \frac{2k(c_1 + c_2 \cos 3\theta)}{(c_3 + k) + (c_3 - k)\cos 3\theta} \quad \text{Lin-Bazant} \quad (26)$$

$$g(\theta) = \frac{(7+2k) - 2(1-k)\sin 3\theta}{9} \quad \text{Yang-Shi} \quad (27)$$

Elliptic function proposed by Willams and Warnke [120] is

$$g(\theta) = \frac{(1-K^2)(\sqrt{3}\cos\theta - \sin\theta)}{(1-K^2)(2+\cos 2\theta - \sqrt{3}\sin 2\theta) + (1-2K)^2} + \frac{(2K-1)\sqrt{(2+\cos 2\theta - \sqrt{3}\sin 2\theta)(1-K^2) + 5K^2 - 4K}}{(1-K^2)(2+\cos 2\theta - \sqrt{3}\sin 2\theta) + (1-2K)^2} \quad (28)$$

Hyperbolic function proposed by Yu-Liu [131] is

$$g(\theta) = \frac{2(1-K^2)\cos\theta + (2K-1)\sqrt{4(1-K^2)\cos^2\theta + 5K^2 - 4K}}{4(1-K^2)\cos^2\theta + (K-2)^2} \quad (29)$$

$$F = a\tau_8^2 + b\tau_8 + c\sigma_1 + d\sigma_8 = 1 \quad \text{Chen [31]} \quad (30)$$

$$F = \tau_8^2 + c_1 P(\theta) \tau_8 + c_2 \sigma_8 = C, \quad \text{Podgorski [132]} \quad (31)$$

where

$$P = \cos[1/3 \arccos(\cos 3\theta) - \beta] \\ F = (\alpha \tau_8)^2 + m[b\tau_8 P(\theta, \lambda) + c\sigma_8] = C \quad \text{Menetrey-Willam [133]} \quad (32)$$

$$F = J_3 + cJ_2 - (1-\eta)c^3 = 0, \quad \text{Krenk [134]} \quad (33)$$

Some other failure criteria were proposed as follows:

$$F = (\sqrt{3}/\sqrt{2})\tau_8 - k(1 - 3\sigma_8/\sigma_{ttt})^\alpha - (3\sigma_8/\sigma_{ccc})^\beta = 0 \quad \text{Yu-Liu [44]} \quad (34)$$

where α and β are the shape functions, $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$.

$$F = \tau_8 + a(1-\lambda) \left(\frac{\sigma_8 + b}{\sigma_8 + 2a} \right)^\alpha + a\lambda \left(\frac{\sigma_8 + b}{\sigma_8 + 3a} \right)^\beta \quad \text{Qu [44]} \quad (35)$$

where $\lambda = (\sin^3 \theta)^{(0.8 + 3/2 + \theta)}$

$$F = \frac{\sigma_m}{1 - (\eta/\eta_0)^n} \quad \text{Shen [135]} \quad (36)$$

where

$$\eta = \frac{1}{\sqrt{2}} \left[\left(\frac{\tau_{12}}{\sigma_{12}} \right)^2 + \left(\frac{\tau_{13}}{\sigma_{13}} \right)^2 + \left(\frac{\tau_{23}}{\sigma_{23}} \right)^2 \right]^{1/2} \\ F = J_2 - a + bI_1 J_3^{1/3} = C, \quad \text{Yin-Li et al [136]} \quad (37)$$

$$F = \tau_8 - a \left(\frac{b - \sigma_8}{c - \sigma_8} \right)^d \quad \text{Guo-Wang [137,138]} \quad (38)$$

where $c = c_t(\cos 3\theta/2)^{1.5} + c_c(\sin 3\theta/2)^{1.5}$,

$$F = a\tau_8^{1.5} + b\tau_8 \cos \theta + a\sigma_8 = C,$$

$$\text{Zhang-Huang [139]} \quad (39)$$

$$F = a\tau_8^2 + (b + c \cos \theta)\tau_8 + d\sigma_8 = C, \quad \text{Jiang [44,140]} \quad (40)$$

$$F = \tau_{8t} + a_1\sigma_8 + b_1\sigma_8^2 = C_1, \quad (\theta = 0^\circ)$$

$$F' = \tau_{8c} + a_2\sigma_8 + b_2\sigma_8^2 = C_2, \quad (\theta = 60^\circ)$$

$$\text{Song-Zhao [141,142]} \quad (41)$$

$$\text{where } \tau_8(\theta) = \tau_8 \cos^2(3\theta/2) + \tau_8 \sin^2(3\theta/2)$$

$$F = \tau_8^2 + (A\sigma_8 + B)[1 - (1 - c)(1 - \cos 3\theta)] = C,$$

$$\text{Genev-Kissiyuk [27]} \quad (42)$$

$$F = \tau_8 + A\sigma_8 \sqrt{1 - B \cos 3\theta} = C, \quad \text{Gudehus [127]} \quad (43)$$

Interested readers are referred to reviewing literatures written by WF Chen [31] and Shen-Yu [52]. Some failure surfaces with cross section of quadratic curve and regular triangle were derived from hypo-elasticity by Tokuoka [143]. Two J_3 -modified Drucker-Prager criteria were proposed by Lee and Ghosh in [144]. Another modified von Mises criterion proposed by Raghava-Cadell for polymers was used for viscoplastic analysis by Hu, Schimit, and Francois in [145]. Other yield criteria joining all the three invariants were proposed by Hashiguchi in [146], Maitra-Majumdar in [147], Haddow-Hrudey in [148] *et al*.

3.2.4 Octahedral shear cap model

Drucker *et al* was upon the first to suggest that soil might be modeled as an elasto-plastic work-hardening material (see Section 6.3 of this article). They proposed that successive yield functions might resemble Drucker-Prager cones with convex end caps. Based on the same idea of using a cap as part of the yield surface, various types of cap models have been developed at Cambridge University. They are referred to as the Cam-clay model and the Modified Cam-clay model. Discussions of various Cambridge models were summarized by Parry [149], Palmer [31], and Wood [150].

Multi-parameter criterion of SSS theory takes three principal shear stresses and the hydrostatic stress into account. They are the curved failure surfaces mediated between the failure surface of the SSS theory (Single-shear strength theory) and the failure surface of the TSS theory (Twin-shear strength theory) proposed and developed in China from 1961 to 1990 as shown in Fig. 1.

According to Eq. (11), all the failure criteria of OSS series strength theory can be expressed in terms of three principal stress τ_{13} , τ_{12} , and τ_{23} . So, this series of strength theory may be also referred to as the three-shear strength theory [46].

3.2.5 Applications of the OSS theory

The octahedral-shear stress yield criterion (von Mises criterion) has been widely used for metallic materials. Octahedral-shear failure criterion (Drucker-Prager criterion) and the octahedral-shear cap model were used in soil mechanics and geotechnological engineering, and implemented

into nonlinear FEM codes. Various multi-parameter octahedral shear failure criteria were used for concrete.

It is very interesting, as indicted by Zyczkowski [30], that various expressions of SSS equations and OSS equations in general form (13) or its particular cases were later repeated—more or less independently—by many authors [30]. If we utilize Eq. (11), many expressions are the same.

3.3 Twin-shear strength theory (TSS theory)

It is clear that there are three principal shear stresses τ_{13} , τ_{12} , and τ_{23} in a stressed element. However, τ_{13} , τ_{12} , and τ_{23} are not independent, there are only two independent components in three principal shear stresses because the maximum principal shear stress τ_{13} equals the sum of the other two, ie, $\tau_{13} = \tau_{12} + \tau_{23}$. So, the idea of *twin-shear* was introduced and developed by Yu and Yu *et al* in 1961-1990 [152–160].

This series of strength theories considers the maximum principal shear stress τ_{13} and intermediate principal shear stress τ_{12} (or τ_{23}), and the influence of the normal stresses σ_{13} and σ_{12} (or σ_{23}) acting on the same section, respectively. It is referred to as the twin-shear strength theory (TSS theory), and can be written mathematically as

$$F[\tau_{13}, \tau_{12}; \sigma_{13}, \sigma_{12}] = C,$$

$$\text{when } f(\tau_{12}, \sigma_{12}) \geq f(\tau_{23}, \sigma_{23}) \quad (44)$$

$$F'[\tau_{13}, \tau_{23}; \sigma_{13}, \sigma_{23}] = C,$$

$$\text{when } f(\tau_{12}, \sigma_{12}) \leq f(\tau_{23}, \sigma_{23}) \quad (44')$$

The first criterion in the TSS category was originally postulated in 1961, and has since developed into a new series of strength theory. Among the main stream are the twin-shear yield criterion (one-parameter) [151,152], the generalized twin-shear strength theory (two-parameter) [155], the twin-shear ridge model [131], the twin-shear multiple-slip condition for crystals [157], the multi-parameter twin-shear criterion [158,159], and the twin-shear cap model [160]. The systematical theories and their applications were summarized in a new book [156].

3.3.1 Twin-shear yield criterion (Yu 1961)

This is a one-parameter criterion of the twin-shear strength theory. The idea and expressions of twin-shear yield criterion are as follows [152,153]:

$$F = \tau_{13} + \tau_{12} = \sigma_1 - \frac{1}{2}(\sigma_2 + \sigma_3) = \sigma_s,$$

$$\text{when } \sigma_2 \leq \frac{\sigma_1 + \sigma_3}{2} \quad (45)$$

$$F = \tau_{13} + \tau_{23} = \frac{1}{2}(\sigma_1 + \sigma_2) - \sigma_3 = \sigma_s,$$

$$\text{when } \sigma_2 \geq \frac{\sigma_1 + \sigma_3}{2} \quad (45')$$

Twin-shear yield criterion is a special case of the Twin-shear strength theory [155].

The generalization of the Twin-shear yield criterion by adding a hydrostatic stress term was given by Yu in 1962

[154]. The limit surface is a pyramid with regular hexagonal cross section similar to the yield locus of the twin-shear yield criterion. The expression is

$$F = \tau_{13} + \tau_{12} + \beta \sigma_m = C \quad F = \tau_{13} + \tau_{23} + \beta \sigma_m = C \quad (46)$$

3.3.2 Twin-shear strength theory (Yu-He 1985)

A new and very simple strength theory for geomaterials was proposed by Yu *et al* [155]. This is a two-parameter criterion of the twin-shear strength theory. The idea and mathematical modeling are as follows:

$$F = \tau_{13} + \tau_{12} + \beta(\sigma_{13} + \sigma_{12}) = C, \quad \text{when } \tau_{12} + \beta \sigma_{12} \geq \tau_{23} + \beta \sigma_{23} \quad (47)$$

$$F = \tau_{13} + \tau_{23} + \beta(\sigma_{13} + \sigma_{23}) = C, \quad \text{when } \tau_{12} + \beta \sigma_{12} \leq \tau_{23} + \beta \sigma_{23} \quad (47')$$

It can be expressed in terms of three principal stresses as follows:

$$F = \sigma_1 - \frac{\alpha}{2}(\sigma_2 + \sigma_3) = \sigma_t, \quad \text{when } \sigma_2 \leq \frac{\sigma_1 + \alpha \sigma_3}{1 + \alpha} \quad (48)$$

$$F' = \frac{1}{2}(\sigma_1 + \sigma_2) - \alpha \sigma_3 = \sigma_t, \quad \text{when } \sigma_2 \geq \frac{\sigma_1 + \alpha \sigma_3}{1 + \alpha} \quad (48')$$

The SD effect and the effect of hydrostatic stress are taken into account in the twin-shear strength theory. The limit surface of the twin-shear strength theory is a hexagonal pyramid whose cross sections (in the π plane) are symmetric but non-regular hexagons. It is the upper (external) bound of all the convex limit loci as shown in Fig. 1. No admissible convex limit surface may exceed the twin-shear limit surface.

3.3.3 Verifications of the twin-shear strength theory

The verifications of the TSS theory were given by Li-Xu *et al* [161] and Ming-Sen-Gu [162] by testing the Laxiwa granite of a large hydraulic power station in China and rock-like materials under true tri-axial stresses. The twin-shear strength theory predicted these experimental results. This conclusion was also given by comparing the experimental data of Launay-Gachon's tests [163] for concrete and other experimental data [164–166]. The experimental results of Winstone [167] agreed well with the twin-shear yield criterion.

3.3.4 Twin-shear multi-parameter criteria

The twin-shear strength theory can also be extended into various multi-parameter criteria for more complex conditions. The expressions are as follows [158,159]:

$$F = \tau_{13} + \tau_{12} + \beta_1(\sigma_{13} + \sigma_{12}) + A_1 \sigma_m + B_1 \sigma_m^2 = C \quad (49)$$

$$F = \tau_{13} + \tau_{23} + \beta_2(\sigma_{13} + \sigma_{23}) + A_2 \sigma_m + B_2 \sigma_m^2 = C \quad (49')$$

where β, A, B, C are the material parameters.

3.3.5 Twin-shear cap model

Twin-shear cap model was proposed by Yu and Li [160]. It has been implemented into an elasto-plastic FE program.

3.3.6 Applications of the TSS theory

TSS theory (twin-shear series of strength theory) has pushed the strength theory study to a new advance by forming the upper (outer) bound for all the possible convex failure surfaces coincided with the Drucker postulation on the deviatoric plane in stress space as shown in Fig. 1.

The twin-shear yield criteria had been used successfully in the plane strain slip line field by Yu-Liu [168], plane stress characteristic field by Yan-Bu [169–171], metal forming by Zhao *et al* [172–176], and limiting analysis of structures by Li [177], Huang-Zeng [178], JJ Chen [179], Wang [180], etc. TSS theory was implemented into some finite element programs by An *et al* [181], by Quint Co [182–184], etc. and applied in elasto-plastic analysis and elasto-visco-plastic analysis of structures by Li-Ishii-Nakazato [185], Luo-Li [186], Li and Ishii [187], and Liu *et al* [188].

TSS theory was also applied in the plasticity of geomaterials by Zhang [189] and Zhu [190], in wellbore analysis [191], in gun barrel design [192], punching of concrete slab [193], and soil liquefaction [194], etc. Some applications of the twin-shear strength theory can be seen in [195–199]. The results of the applications indicate that it could raise the bearing capacities of engineering structures more than the Mohr-Coulomb's, ie, improves on the conservative Mohr-Coulomb's. So, there is a considerable economical benefit by using the new approach if the strength behavior of materials is adaptive to the twin shear theory [156,185].

Three series of strength theories, ie, SSS series, OSS series, and TSS series are summarized briefly in Table 1.

4 YIELD CRITERIA FOR METALLIC MATERIALS

Many articles and books, as indicated in Table 1, have reviewed this object. We will supplement the maximum deviatoric stress criterion, or twin shear criterion, and the unified yield criterion in Section 5. Some experimental data are summarized as shown in Table 2.

The experimental results are taken from Guest [62], Scoble [200,201], Smith [202], Lode [203], Taylor-Quinney [204], Ivey [205], Paul [18], Bell [22], Michno-Findley [25], Pisarenko-Lebedev [206], and others [207–219]. The discrepancies among different experiments and different materials are large. Up to now, none of the above yield criteria agreed with the experiments for different materials. After the comparison of the shear yield strength and tensile yield strength for thirty materials, Kishkin and Ratner [220] divided the metals into four kinds according to the ratio of the shear yield strength with tensile yield strength τ_s/σ_s as follows:

- $\tau_s/\sigma_s < 0.40$ (0.31–0.41, eight materials). It is a non-convex result, no yield criterion agreed
- $\tau_s/\sigma_s \approx 0.50$ (0.48–0.53, five materials). It is agreed with the single-shear yield criterion (Tresca yield criterion)
- $\tau_s/\sigma_s \approx 0.58$ (0.54–0.62, nine materials). It is agreed with the tri-shear yield criterion (von Mises yield criterion)
- $\tau_s/\sigma_s \approx 0.68$ (0.67–0.71, eight materials). It is agreed with the twin-shear yield criterion.

Four sets of experimental results of Winstone [167] show that the initial yield surfaces indicated a ratio of shear yield

Table 2. Comparison of three yield criteria with experimental results

Researchers	Materials	Specimen	shear/tension τ_y/σ_y	Suitable Criterion
Guest, 1900	steel,		0.474, 0.727	
Guest, 1900	steel, brass, etc.	tubes	0.474	
Hancock, 1906,1908	mild steel	solid rods,tube	0.50 to 0.82	
Scoble, 1906	mild steel	solid rods	0.45 to 0.57	Tresca
Smith, 1909	mild steel	solid rods	0.55-0.56	between Tresca and von Mises
Turner, 1909-1911	steels		0.55 to 0.65	Tresca
Mason, 1909	mild steel	tubes	0.5 0.64	Tresca
Scoble,1910	steel		0.376, 0.432, 0.452	no one agreed
Becker, 1916	mild steel	tubes		no one agreed
Seeley and Putnam	steels	bars & tubes	0.6	> von Mises
Seigle and Cretin, 1925	mild steel	solid bars	0.45 to 0.49	Tresca
Lode, 1926	iron, copper etc.	tubes		von Mises
Ros and Eichinger, 1926	mild steel	tubes		von Mises
Taylor and Quinney, 1931	aluminum, steel	tubes		von Mises
	copper, mild steel			>von Mises
Morrison, 1940	mild steel	tubes		Tresca, von Mises
Davis, 1945-1948	copper,	tubes		von Mises
Osgood, 1947	aluminum alloy	tubes		von Mises
Cunningham, 1947	magnesium alloy	tubes		von Mises
Bishop-Hill, 1951	polycrystals	tubes	0.54	von Mises
Fikri, Johnson, 1955	mild steel	tubes		> von Mises
Marin and Hu, 1956	mild steel	tubes		von Mises
Naghdi, Essenberg, and Koff, 1958	aluminum alloy	tubes		> von Mises
Hu and Bratt, 1958	aluminum alloy	tubes		von Mises
Ivey, 1961	aluminum alloy	tubes		Twin shear
Bertsch-Findley, 1962	aluminum alloy	tubes		von Mises
Mair-Pugh, 1964	copper	tubes		Twin shear
Mair-Pugh, 1964	copper	tubes		von Mises
Chernyak <i>et al</i> , 1965	mild steel	tubes		von Mises
Miastkowski, 1965	brass			von Mises
Rogan, 1969	steel	tubes		Tresca
Mittal, 1969, 1971	aluminum	tubes	0.57	von Mises
Dawson, 1970	polycrystals		0.64	near Twin shear
Phillips <i>et al</i> , 1968- 1972	aluminum	tubes at elevated temperature		between Tresca and von Mises
Deneshi <i>et al</i> , 1976	aluminum, copper	tubes, low temperature	0.6	> von Mises
Pisarenko <i>et al</i> , 1984	copper, Cr-steel	tubes, low temperature		von Mises
Winstone, 1984	nickel alloy	tubes at elevated temperature	0.7	Twin shear
Ellyin, 1989	titanium	tubes	0.66-0.7	Twin shear
Wu-Yeh, 1991	Aluminum	tubes	0.58	von Mises
	stainless steel		0.66-0.7	Twin shear
Ishikawa, 1997	stainless steel	tubes	0.6-0.63	> von Mises
Granlund,Olsson, 1998	structural steel	flat cruciform specimens		between Tresca and von Mises

stress to tensile yield stress of 0.7. He said: "The ratio of torsion shear yield stress to tensile yield stress is $\cong 0.7$, surprisingly high when compared with the values of 0.58 and 0.5 expected from the von Mises and Tresca yield criteria, respectively. Clearly neither of these criteria can accurately model the bi-axial yield behavior of Mar-M002 castings." [167]

The Tresca and von Mises yield criteria excepted, Haythornthwaite proposed a new yield criterion [90]. This new yield criterion is referred to as the maximum reduced stress (deviatoric stress) S_{\max} yield criterion

$$f = S_{\max} = 1/3(2\sigma_1 - \sigma_2 - \sigma_3) = 2/3\sigma_s \quad (50)$$

The similar idea of maximum deviatoric stress criterion may be traced back to the deviatoric strain (shape change) by Schmidt in 1932 [30] and Ishlinsky in 1940 [222], and the linear approximation of the von Mises criterion by Hill in 1950 [18,223], then first proposed independently from the idea of maximum deviatoric stress by Haythornthwaite in 1961 [90]. The expression of Hill was

$$f = (2\sigma_1 - \sigma_2 - \sigma_3) = m\sigma_s \quad (51)$$

Comments to this criterion were made by Paul [18] and by Zyczkowski [30].

Another new idea was proposed by Yu [152]. It assumed that yielding begins when the sum of the two larger principal shear stresses reaches a magnitude C . It is clear that only two principal shear stresses in three principal shear stresses τ_{13} , τ_{12} , and τ_{23} are independent variables, because of the maximum principal shear stress τ_{13} equals the sum of the other two. So, it is referred to as the twin-shear stress yield criterion that is given in Eqs. (44) and (44').

The twin-shear yield criterion can also be introduced from the generalized twin-shear strength theory proposed by Yu in 1985 [155] (when $\alpha = 1$ in Eq. (46)). This yield surface is the upper (outer) bound of all the convex yield surfaces.

All the yield criteria, including the Tresca, von Mises, and twin-shear yield criterion are single-parameter criterion. Many researchers followed Bridgman [224–228] and assumed that materials are hydrostatic pressure independent. Numerous experiments carried out by Bridgman (1882–1961) at Harvard proved that the yielding of metals is unaffected by hydrostatic pressure. His experiments included thirty metals. Most non-metallic materials, however, are hydrostatic pressure dependent [229–233].

The yield criteria had been used successfully in the plane strain slip line field by Hencky [234], Geiringer [235], Prandtl [236–238], Prager [239,240], Johnson [241], Yu *et al*

[168], and others. The uses of yield criterion in plane stress characteristic field and axisymmetric characteristic field and metal forming can be seen in Kachanov [242], Yan-Bu [169–171], Hill [243], Haar-von Karman [244], Sokolovski [245], and Thomsen-Yang-Kobayash, [247], etc. The applications in damage and yield of ductile media with void nucleation can be found in Gurson [248,249], Tvergaard [250–252], Gologanu-Leblond-Perrin-Devaux [253] *et al.* Limiting analysis and elasto-plastic analysis of structures by using the yield criteria can be seen in Drucker [254,255], Hodge [256], and Save *et al* [257]. The implementations of various yield criteria to variety finite element programs were summarized by Brebbia in [258]. However, how to choose a reasonable yield criterion is an important problem.

It is still a problem to find a unified yield criterion that can be applicable to more than one kind of material and establish the relationship among various yield criteria.

5 UNIFIED YIELD CRITERIA

5.1 Curved general yield criteria

5.1.1 Curved general yield criterion between SS and OS yield criteria

A curved general yield criterion lying between SS (Single-shear, Tresca) and OS (Octahedral-shear, von Mises) criteria was proposed by Hershey in 1954 [259], Davis in 1961 [260,261], Paul in 1968 [18], Hosford in 1972 [262], Barlat-Lian in 1989 [263], and explained by Owen & Peric [264] *et al* as follows:

$$f_1 = (S_1 - S_2)^{2k} + (S_2 - S_3)^{2k} + (S_3 - S_1)^{2k} = 2\sigma_s^{2k}$$

$$\text{or } f_1 = |\sigma_2 - \sigma_3|^m + |\sigma_3 - \sigma_1|^m + |\sigma_1 - \sigma_2|^m = 2\sigma_s^{2k} \quad (52)$$

This expression is a generalization of Bailey's flow rule [260] for combined stress creep by Davis [261] as a yield surface lies inside the von Mises yield criterion and outside the Tresca yield criterion. This kind of yield criteria is sometimes called the Bailey-Davis yield criterion.

5.1.2 Curved general yield criteria between OS and TS yield criteria

The curvilinear general yield criteria lying between the OS yield criterion (octahedral shear yield criterion) and the TS yield criterion (twin-shear yield criterion) were proposed by Tan in 1990 [265] and Karafillis & Boyca in 1993 [266]. The expression is

$$f = S_1^{2k} + S_2^{2k} + S_3^{2k} = \frac{2^{2k} + 2}{3^{2k}} \sigma_s^{2k} \quad (53)$$

5.1.3 Curved general yield criteria between SS and TS yield criteria

Tan [265] and Karafillis & Boyca [266] obtain a general yield criterion lying between the lower bound (SS yield criterion) and the upper bound (TS yield criterion) yield criterion as follows:

$$\phi = (1 - c)f_1 + c \frac{3^{2k}}{2^{2k-1} + 1} f_2, \quad c \in [0, 1] \quad (54)$$

5.1.4 Drucker criterion

Edelman and Drucker suggested the following criterion [82]

$$J_2^3 - C_d J_3^2 = F \quad (55)$$

Dodd-Narusec [267] extended this equation in the following expression

$$(J_2^3)^m - C_d (J_3^2)^m = F^m \quad (56)$$

It is a series of curved yield criteria ($m=1$ or $m=2$) lying outside the SS (Tresca) yield criterion.

5.1.5 Hosford criterion

Hosford [262] proposed a criterion as follows

$$[\frac{1}{2}(\sigma_1 - \sigma_2)^m + \frac{1}{2}(\sigma_1 - \sigma_3)^m]^{1/m} = \sigma_s \quad (57)$$

A series of yield criteria can be given when $m=1$ to $m=\infty$.

5.1.6 Simplification of anisotropic yield criterion

Hill proposed a new yield criterion as follows [268]

$$f|\sigma_2 - \sigma_3|^m + g|\sigma_3 - \sigma_1|^m + h|\sigma_1 - \sigma_2|^m + C|2\sigma_1 - \sigma_2 - \sigma_3|^m + |2\sigma_2 - \sigma_1 - \sigma_3|^m + |2\sigma_3 - \sigma_1 - \sigma_2|^m = \sigma_s^m \quad (58)$$

where $m \geq 1$, the six parameters f , g , h , a , b , and c are constants characterizing the anisotropy. For the isotropic case, $f=g=h$, $a=b=c$, it is three-parameter criterion. Dodd and Naruse [267] take $f=g=h=1$, from which it follows that

$$f = |\sigma_2 - \sigma_3|^m + |\sigma_3 - \sigma_1|^m + |\sigma_1 - \sigma_2|^m + C|2\sigma_1 - \sigma_2 - \sigma_3|^m + |2\sigma_2 - \sigma_1 - \sigma_3|^m + |2\sigma_3 - \sigma_1 - \sigma_2|^m \sigma_s^m \quad (58')$$

A series of curved yield criteria between SSS criterion and TSS criterion can be given when $m=1$ to $m=\infty$. Similar yield criteria can also be introduced from the anisotropic yield criteria of Hosford [262] and Barlat *et al* [263,269,270,735].

All the generalized yield criteria mentioned above are smooth, convex, and curvilinear yield criteria lying between single-shear and twin-shear yield criteria. They are the non-linear unified yield criteria. However, they are not convenient to use in the analytic solution of elasto-plastic problems.

5.2 Linear unified yield criterion

5.2.1 Unified yield criterion

A new linear unified yield criterion was introduced from the unified strength theory by Yu-He [271–274]. The mathematical modeling is

$$f = \tau_{13} + b\tau_{12} = C, \quad \text{when } \tau_{12} \geq \tau_{23} \quad (59)$$

$$f' = \tau_{13} + b\tau_{23} = C, \quad \text{when } \tau_{12} \leq \tau_{23} \quad (59')$$

where b is a coefficient of the effect of the other principal shear stresses on the yield of materials. The unified yield criterion can be expressed in terms of three principal stresses as follows

$$f = \sigma_1 - \frac{1}{1+b}(b\sigma_2 + \sigma_3) = \sigma_s, \text{ when } \sigma_2 \leq \frac{1}{2}(\sigma_2 + \sigma_3) \quad (60)$$

$$f' = \frac{1}{1+b}(\sigma_1 + b\sigma_2) - \sigma_3 = \sigma_s, \text{ when } \sigma_2 \geq \frac{1}{2}(\sigma_2 + \sigma_3) \quad (60')$$

It is a linear unified yield criterion. It contains two families of yield criterion: one is the convex unified yield criterion lying between the single-shear and twin-shear yield criteria (when $0 \leq b \leq 1$). Another is the non-convex yield criterion lying outside the twin-shear yield criterion (when $b > 1$) or lying inside the single-shear yield criterion (when $b < 0$). So, it can be predicted to most results listed in Table 2.

This unified yield criterion encompasses the single-shear (Tresca) yield criterion (when $b=0$), twin-shear yield criterion (when $b=1$), and the octahedral-shear yield criterion (von Mises) as special cases or linear approximation ($b=1/2$). A lot of new linear yield criteria can be also introduced [272]. It can be adopted for all the metallic materials with the same yield strength both in tension and compression. The linear unified yield criterion is also a special case of the unified strength theory proposed by Yu in 1991 [271,272], which will be described in Section 7 of this article.

The varieties of the yield loci of this unified yield criterion at the π plane and plane stress are shown in Fig. 2 and Fig. 3.

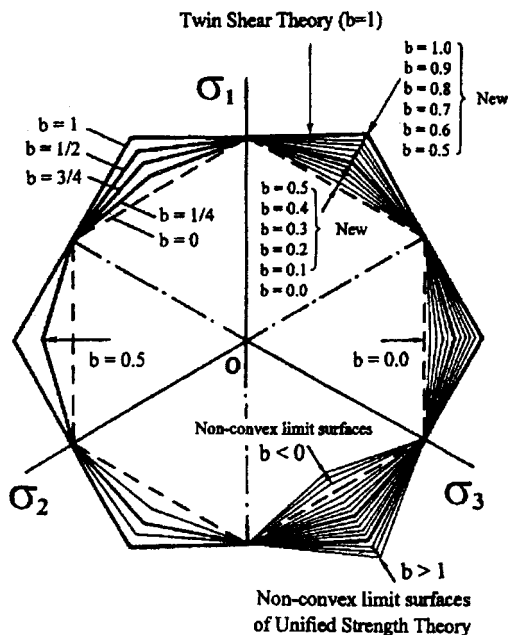


Fig. 2 Varieties of the unified yield criterion in π plane [271,272]

5.2.2 Extension of the unified yield criterion

The relationship between shear yield stress τ_s and tensile yield stress σ_s of materials can be introduced from the unified yield criterion as follows

$$\tau_s = \frac{1+b}{2+b} \sigma_s \quad \text{or} \quad \alpha_\tau = \frac{\tau_s}{\sigma_s} = \frac{1+b}{2+b} \quad (61)$$

It is shown that: (1) The shear strength of ductile materials is lower than tensile strength; (2) Yield surfaces are convex when $0 \leq b \leq 1$ or $1/2 \leq \alpha_\tau \leq 2/3$; (3) Yield surfaces are non-convex when $b < 0$ and $b > 1$; or the ratio of shear yield stress to tensile yield stress $\alpha_\tau < 1/2$ and $\alpha_\tau > 2/3$.

5.2.3 Non-convex yield criterion

Non-convex yield criterion was merely investigated before. The unified yield criterion can be used to describe the results of $\tau_s/\sigma_s < 0.5$ ($b < 0$) and $\tau_s/\sigma_s > 2/3$ ($b > 1$). It is a non-convex result. Recently, Wang and Dixon [275] proposed an empiric failure criterion in $(\sigma-\tau)$ combined stress state. It can be fitted in with those experimental results in $(\sigma-\tau)$ combined stress state of Guest [62], and Scoble [200,201] with $\tau_s/\sigma_s = 0.376, 0.432, 0.451$, and 0.474 . It is easy to match these results by using the unified yield criterion with $b = -0.4, b = -0.24, b = -0.18$, and $b = -0.01$, respectively.

5.2.4 Applications of the unified yield criterion

The linear unified yield criterion is convenient to use in the analytical solution of elasto-plastic and other problems. The unified solutions of simple-supported circular plate were given by Ma-He [276] and Ma-Yu *et al* [277,278]. Ma, Yu, and Iwasaki *et al* also gave the unified elasto-plastic solution of rotating disc and cylinder by using the unified yield criterion [279]. The unified solutions of limiting loads of rectangular plate and oblique plates were obtained by Zao *et al* [280] and Li, Yu, and Xiao [281], respectively.

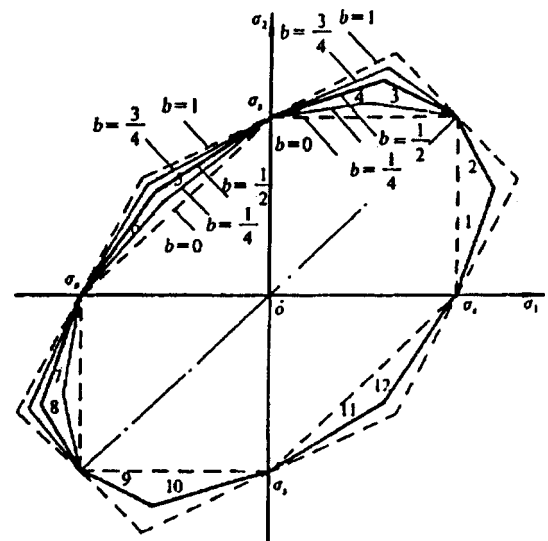


Fig. 3 Varieties of the unified yield criterion in plane stress [271,272]

The further studies of limit speeds of variable thickness discs using the unified yield criterion were given by Ma, Hao, and Miyamoto in 2001 [282]. The plastic limit analyses of clamped and simple-supported circular plates with respect to the unified yield criterion were obtained by Guowei, Iwasaki-Miyamoto *et al* [283], and Ma, Hao *et al* [284,285]. The dynamic plastic behavior of circular plates using the unified yield criterion was studied by Ma, Iwasaki *et al* [286]. Qiang and Xu *et al* [287] gave the unified solutions of crack tip plastic zone under small scale yielding and the limit loads of rectangular plate, etc, respectively, by using the unified yield criterion. A series of results can be introduced from these studies.

Some research results concerning the yield criterion were given in [288–318].

All the yield criteria including the Tresca criterion, von Mises criterion, twin-shear criterion, and the unified yield criterion can be adopted only for those materials with same yield stress in tension and in compression. They cannot be applied to rock, soil, concrete, ice, iron, ceramics, and those metallic materials which have the SD effect (strength difference at tension and compression). The SD effects of high strength steels, aluminum alloys, and polymers were observed in 1970s, eg, Chait [319], Rauch and Leslie [320], Drucker [321], Richmond and Spitzig [322], and Casey and Sullivan *et al* [323]. The SD effect is related to the effect of the hydrostatic stress. The hydrostatic pressure produces effects increasing the shearing capacity of the materials. Effects of hydrostatic pressure on mechanical behavior and deformation of materials were summarized recently by Lewandowski and Lowhaphandu [324]. The effects of hydrostatic stress and the SD effect of metals, rock, polymers, etc were summarized recently by Yu in 1998 [156].

The generalized failure criteria considering the SD effect and the influence of hydrostatic stress have to be used. The limit loci of the generalized failure criteria considering the SD effect and the influence of hydrostatic stress are shown in Fig. 1. The Mohr-Coulomb strength theory [70] and the Yu's twin-shear strength theory [155] are two bounds of all the convex criteria.

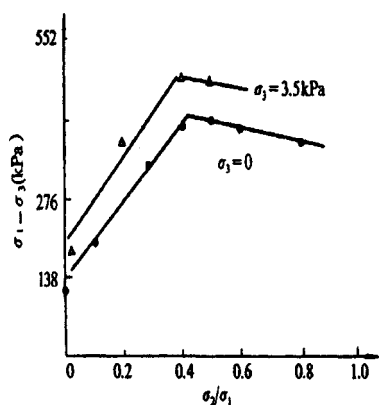


Fig. 4 Effect of the intermediate principal stress [355]

6 FAILURE CRITERIA FOR SPECIFIC MATERIALS

The development of strength theories is closely associated with that of the experimental technology for testing materials in complex stress states. A considerable account of triaxial stress tests were done in the 20th century. There are two kinds of triaxial tests.

In most laboratories, for triaxial tests, cylindrical rock and soil specimens are loaded with an axial stress $\sigma_z = \sigma_1$ (or $\sigma_z = \sigma_3$), and a lateral pressure $\sigma_2 = \sigma_3$ (or $\sigma_2 = \sigma_1$); both can be varied independently, but always $\sigma_2 = \sigma_3$ (or $\sigma_2 = \sigma_1$). The first research seems to be due to Foppl [64], von Karman [71], and Boker [72]. Von Karman and Boker were supervised under Prandtl. Today such tests are done in all rock mechanics and soil mechanics laboratories. This kind of test, unfortunately, is usually called the triaxial test, although it involves only very special combinations of triaxial stress. It is better to refer to this test as the confined compression test, since it is a compression test with a confining lateral pressure [18]. Sometimes it is called the untrue triaxial test or false triaxial test. All the combinations of complex stresses in a confined compression test lie on a special plane in stress space. So, most triaxial tests are only a plane stress test.

In 1914, Boker retested the type of marble used by von Karman in a confined pressure test in which the lateral pressure was the major principal stress. The corresponding Mohr's envelope did not agree with von Karman's (in von Karman's tests, the axial pressure exceeded the lateral pressure). This means that the Mohr-Coulomb criterion could not fit the data adequately in the range of low hydrostatic pressure, although the more general hexagonal pyramid criterion was not ruled out [18,72]. It is evident that the confined compression test is not capable of proving that the intermediate stress is of no influence on the failure criterion.

Another is seldom a true tri-axial test, in which all three principal stresses can be varied independently. Several workers have designed specialized equipment for conducting true triaxial test, eg, Shibata-Karube [91], Mogi [92,93,325–328], Launay-Gachon [163], Desai *et al* [333], Hunsche [336],

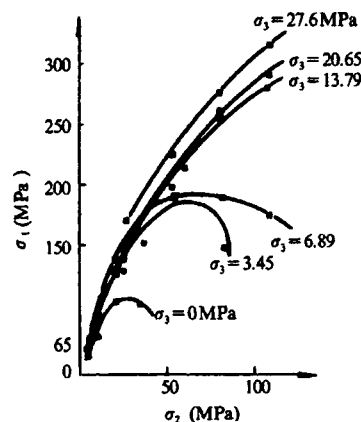


Fig. 5 Effect of the intermediate principal stress [98,99]

Ming *et al* [162], Xu-Geng [349], Michelis [98,99], Li, Xu, and Geng [350], Mier [348], and Wawersik, Carson *et al* [345].

A great amount of effort was dedicated to the development of true-triaxial testing facilities, and the facilities were then used to test engineering materials. Some representative efforts were seen on rocks at Tokyo University and others, on soil at Cambridge University, Karlsruhe University, Kyoto University, and others, and on concrete in Europe and the United States.

Mogi's persistent effort revealed that rock strength varied with the intermediate principal stress σ_2 , which was quite different from what had been depicted in the conventional Mohr-Coulomb theory. The study was further extended [346,353,354] to a understanding that the σ_2 effect had two zones: the rock strength kept on increasing, when σ_2 built up its magnitude from σ_3 , until reaching a maximum value; beyond that, the rock strength decreased with the further increase of σ_2 . Xu and Geng also pointed out that varying σ_2 , only, while keeping the other principal stresses σ_1 and σ_3 unchanged, could lead to rock failure, and this fact could also be attributable to inducing earthquakes [347]. Michelis indicated that the effect of intermediate principal stress is the essential behavior of materials [98,99]. Figures 4 and 5 are taken from Mazanti-Sowers [355] and Michelis [98,99]. The effect of the intermediate principal stress on rock is evident.

On a modified high pressure true tri-axial test facility, Li and his colleagues [161,350] tested some granite and showed that the σ_2 effect is significant. This result is consistent with the twin-shear strength theory. Ming and his coworkers [162] and Lu [358,359] also reached the same conclusion.

Most true tri-axial tests are three compressive stresses. Triaxial tension-compression tests for multiaxial stresses were done by Ming, Shen, and Gu [162], and by Calloch and Marquis [344]. A true triaxial cell for testing cylindrical rock specimens was developed by Smart *et al* [341–343]. This true triaxial cell can be suited for testing cylindrical rock specimens.

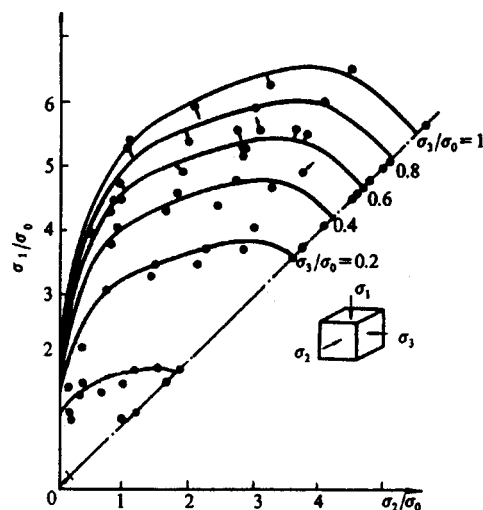


Fig. 6 Effects of the intermediate principal stress of concrete [163]

Recently, at the University of Wisconsin, a new true tri-axial testing system was designed, calibrated, and successfully tested by Haimson and Chang [356]. It is suitable for testing strong rocks, which emulates Mogi's original design [93] with significant simplifications. Its main feature is very high loading capacity in all three orthogonal directions, enabling the testing to failure of hard crystalline rocks subjected to large minimum and intermediate principal stresses [346].

A mathematical proof regarding the twin-shear theory and the single-shear theory was given by citing the mathematical concept of convex sets [360,361]. It is shown that the twin shear strength theory is the exterior (upper) bound and the single-shear theory is the interior (lower) bound of all the convex limiting loci on the π plane as shown in Fig. 1.

The true tri-axial tests on concrete bear many similarities with that on rocks, both in testing facilities and test results. Many such tests have been reported by researchers in France, Japan, Germany, the former Soviet Union, the United States, and China.

Through numerous true tri-axial tests on both rock and concrete, the existence of the σ_2 effect has now been well recognized as characteristic of these materials [90–99, 137–142, 325–328, 346–373]. Figure 6 shows the effects of the intermediate principal stress of concrete given by Launary-Gachon in 1972 [163]. The effects of the intermediate principal stress of metals, rock, concrete, etc were summarized by Yu in 1998 [156].

In the United States, an enhancement factor was introduced in the ACI Standard [362] guiding designs of pre-stressed concrete pressure vessels and safety shells for nuclear power station as shown in Fig. 7.

This standard and many experimental results allow higher permissible strength to be used in concrete and in rock under triaxial compression stress states, and hence lead to higher economical effectiveness in construction use. More importantly, the impact of the concept is expected to be enormous to the design of ordinary engineering structures.

The wider application of the enhancement-factor concept on a global scale is, on one hand, to bring tremendous energy saving and pollution curbing; it calls, on the other hand, for

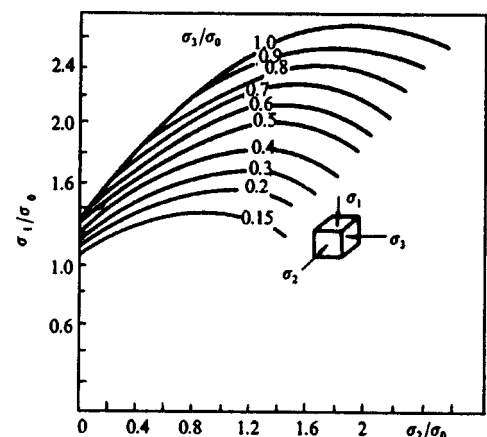


Fig. 7 Enhanced σ_2 effect in concrete strength [362]

a theoretical support on which the concept could be based. The engineering practice in general has a desire to have a new strength theory, which should be more rational and more consistent to the experimental data than what can be done by the Mohr-Coulomb single-shear strength theory. Some failure criteria (Tresca, von Mises, Mohr-Coulomb, and maximum tensile stress theory) were reviewed by Shaw [35].

6.1 Failure criteria for rock

Up to now, more than 20 strength (yield or failure) criteria for rocks have been developed, but only a few of the criteria are widely used in rock engineering. Failure criteria of rocks were summarized by Jaeger and Cook [363], Lade [122,364], Andreev [45], and Sheorey [50]. Various researches and applications can be found in [363–438].

The Mohr-Coulomb theory is the most widely applied one. Some other nonlinear Mohr-Coulomb criteria similar to the Hoek-Brown criterion were summarized in the literatures of Andrew [45] and Sheorey [50]. The Ashton criterion was extended by JM Hill and YH Wu [365]. All the Mohr-Coulomb, Hoek-Brown, and most kinds of empirical rock failure criteria (Eqs. (4)–(7)) only take the σ_1 and σ_3 into account. They may be referred to as the single-shear strength theory ($\tau_{13} = (\sigma_1 - \sigma_3)/2$). The effects of the intermediate principal stress σ_2 were not taken into account in these criteria (see Eqs. (3)–(7)). The general form of these failure criteria may be expressed as follows:

$$F = f_1(\sigma_1 - \sigma_3) + f_2(\sigma_1 + \sigma_3) + f_3(\sigma_1) = C \quad (62)$$

Some single-shear type failure criteria for rock are given in section three (Eqs. (4)–(7)), the other two failure criteria for rock are

$$\sigma_1 - \sigma_3 = \sigma_c + a\sigma_3^b \quad \text{Hobbs [366]} \quad (63)$$

$$\sigma_1 - \sigma_3 = a(\sigma_1 + \sigma_3)^b \quad \text{Franklin [367]} \quad (64)$$

Mogi [92,93,325–328] proposed a combined failure criterion of octahedral shear stresses τ_8 and σ_{13} for rock as follows:

$$F = \tau_8 + A(\sigma_1 + \sigma_3)^n, \quad F = \tau_8 + f(\sigma_1 + \alpha\sigma_2 + \sigma_3) \quad \text{or}$$

$$F = \tau_{13} + \beta\sigma_{13} + A\sigma_m = C, \quad F = \sigma_1 - \alpha\sigma_3 + A\sigma_m = C \quad (65)$$

The von Mises-Schleicher-Drucker-Prager criterion was modified and applied to rock by Aubertin, L Li, Simon, and Khalfi [368].

The strength tests for various rocks under the action of complex stresses were conducted by Foppl [64], Voigt [65], von Karman [71], Boker [72], Griggs [370], Jaeger [371], Mogi [91,92,325–328], Michelis [98,99] *et al*. Many experimental investigations were devoted to the studies on the effect of the intermediate principal stress, eg, Murrell [372], Handin *et al* [375], Mogi [325–328,385], Amadei *et al* [398,406], Kim-Lade [401], Michelis [98,99,409], Desai [403–405], Yin *et al* [136], Gao-Tao [357], Wang, YM Li, and Yu [166], Lu [164,165,358,359], and others. Some octahedral shear type criteria (OSS theory) for rocks were proposed which included the failure criterion for natural polycrystalline rock salt by Hunsche [336,408,410].

According to Wang and Kemeny [416], σ_2 has a strong effect on σ_1 at failure even if σ_3 equals zero. Their polyaxial laboratory tests on hollow cylinders suggested a new empirical failure criterion in which the intermediate principal stress was taken into account. Effect of intermediate principal stress on strength of anisotropic rock mass was investigated by Singh-Goel-Mehrotra *et al* [422]. The Mohr-Coulomb theory was modified by replacing σ_3 by the average of σ_2 and σ_3 . A multiaxial stress criterion for short- and long-term strength of isotropic rock media was proposed by Aubertin, L Li, Simon *et al* [368,369].

Vernik and Zoback [427] found that use of the Mohr-Coulomb criterion in relating borehole breakout dimensions to the prevailing *in situ* stress conditions in crystalline rocks did not provide realistic results. They suggested the use of a more general criterion that accounts for the effect on strength of the intermediate principal stress. Recently, Ewy reported that the Mohr-Coulomb criterion is significantly too conservative because it neglects the perceived strengthening effect of the intermediate principal stress [428].

The twin-shear strength theory was verified by the experimental results of XC Li, Xu, and Liu *et al* [161,350], Gao-Tao [357], and Ming, Sen, and Gu [162]. The comparisons of the twin-shear strength theory with the experimental results presented in literature were given by Lu [164,165,358,359].

The applications of the twin-shear strength theory to rock were given by Li, Xu, and Liu *et al* [350], An, Yu, and X Wu [181], and Luo and ZD Li [186] *et al*. This strength theory has been applied to the stability analyses of the underground cave of a large hydraulic power station at the Yellow River in China [161,350,429,430]. It was also used in the research of the stability of the high rock slopes in the permanent ship-lock at three Gorges on the Yangtze River by Yangtze Science Research Institute in 1997 [432–433], and the analysis of ultimate inner pressure of rings [438]. The twin-shear strength theory was introduced by Sun [436] and Yang [437] to rock mechanics.

The strength criteria of rock joints were described and reviewed by Jaeger [439], Goodman [441], Zienkiewicz-Valliappan-King [442], Barton [439–446], Ghaboussi-Wilson-Isenberg [447], Singh [448,449], Ge [450], Shiryayev *et al* [451], Stimpson [452], Heuze-Barbour [455], Desai-Zaman [456], Lei-Swoboda-Du [459], and recently by Zhao [461,463], WS Chen, Feng, Ge, and Schweiger [464] *et al*. A series of conferences on Mechanics of Joint and Faulted Rock (MJFR) were held [462].

The systematical researches on rock rheology were given by Crestescu [407] and Sun [435].

The threshold associated with the onset of microcracking was seen as a yield criterion for an elasto-plastic model or as a damage initiation criterion in a state variable model. A multiaxial damage criterion for low porosity rocks was proposed by Aubertin and Simon [418].

A monograph on Advanced Triaxial Testing of Soil and Rock was published by the American Society for Testing and Materials (Donaghe, Chaney and Silver, eds [339]).

6.2 Failure criteria for concrete

Failure criteria for concrete including high strength concrete, light concrete, steel fibre concrete, etc were studied by many researchers [465–536]. Various criteria for concrete were proposed by Bresler and Pister [465,466], Geniev [27], Mills-Zimmerman [471], Buyukozturk-Liu-Nilson-Slate, and Tasuji [473], HC Wu [476], Willam-Warnke [120], Ottosen [485], Cedolin-Crutzen-Dei Poli [480], Hsieh, Ting, and WF Chen [492], Dafalias [478], Yang-Dafalias-Herrmann [496], Schreyer-Babcock [499,507], de Boer *et al* [505], Faruque and Chang [508], Jiang [44,140], Song-Zhao-Peng [141,142,516,518], Voyiadjis and Abu-Lebdeh [514,517], Labbane, Saha, and Ting [515], Menetrey-Willam [133], JK Li [523] *et al*

In general, these criteria are the OSS theory (Octahedral-Stress Strength theory) as described above (Eqs. (12)–(44)). WF Chen *et al* [31,41], Zhang [189], and Jiang [44] made a general survey of these criteria. A microplane model for cyclic triaxial behavior of concrete was proposed by Bazant and Ozbolt [512,513]. Recently, a key paper [51] entitled “Concrete plasticity: Past, present, and future” was presented in the *Proceedings of ISSTAD '98* (International Symposium on Strength Theory: Application, Development and Prospects for the 21st Century). The yield criteria of concrete used in concrete plasticity were summarized by WF Chen [51]. Great contributions in the research of fracture and failure of concrete are due to Bazant [79,129,482,486,503] and others. A failure criterion for high strength concrete was proposed by QP Li and Ansari [526,527] at ISSTAD '98. Smooth limit surfaces for metals, concrete, and geotechnical materials was proposed by Schreyer [499,507]. A new book entitled, *Concrete Strength Theory and its Applications*, was published recently [504].

Considerable experimental data regarding the strength of concrete subjected to multi-axial stresses were given, eg, Richart *et al* [229], Balmer [230], Bresler and Pister [465,466], Kupfer *et al* [470], Launay and Gachon [163], Kotsovos and Newman [479], Tasuji *et al* [484], Ottosen [485], Gerstle [489–491], Michelis [98,99], Song and Zhao [141,142,516,518], Wang-Guo [137,138], Traina-Mansor [510], *et al*.

Lu gave some applications of the twin-shear strength theory for concrete under true triaxial compressive state [164,165,358,359]. The solution of the axisymmetrical punching problem of concrete slab by using the twin-shear strength theory was obtained by Yan [193]. The application of the unified strength theory (see next section) to concrete rectangular plate was given by Zao [533].

The strength theory of concrete was also applied to RC (reinforced concrete) and the nonlinear FEM analysis of RC structures by Nilsson [537], Villiappan-Doolan [538], Zienkiewicz-Owen-Phillips [539], Argyris-Faust-Szimmat *et al* [540], Buyukozturk [541], Bathe-Ramaswang [542], Chen [31], Bangash [543], Jiang [44], *et al*. The twin-shear strength theory and the unified strength theory were used in finite element analysis of reinforced concrete beams and plate by Guo-Liang [545], Wang [547], and others.

Damage model for concrete [506,517], multi-axial fatigue [504], bounding surface model [496], blast and hard impact damaged concrete [529], etc were proposed.

6.3 Failure criteria for soils

The behavior of soil under the complex stress state are more complex. Many studies were devoted to these problems since the 1960s [549–627]. In classical soil mechanics, soil problems have generally been solved on the basis of an ideal elastic soil, where the deformation and stability properties are defined by a single value of strength and deformation modules. Sometimes, the Tresca criterion and the von Mises criterion were used. More sophisticated solutions of the bearing capacity problem involving contained plasticity approach reality more closely by use of the elasto-plastic idealization.

The Mohr-Coulomb failure criterion was the most widely used in soil mechanics. However, the failure mechanism associated with this model is not verified in general by the test results, and the influence of the intermediate principal stress is not taken into account. An extended von Mises criterion was proposed by Drucker and Prager in 1952 [117] and now is referred as the Drucker-Prager criterion.

Although it is widely used, the Mohr-Coulomb model does not yield agreement with experimental data for most materials. Furthermore, the great disadvantage of the Mohr-Coulomb model at present is the lack of indication of behavior in the direction of the intermediate principal stress, and it gives far too much deformation. Previous researches of Habib [549], Haythornthwaite (remolded soil) [551,552], Broms and Casbarian [555], Shibata and Karube (clay) [91], Bishop and Green [83,91,95,105,556], Ko and Scott [94], Sutherland and Mesdary [563], Lade and Duncan [122,564], Green [561,562], Gudehus [127,128,588], Matsuoka, Nakai *et al* [121,577,601], Lade and Musante (remolded clay) [97], Vermeer [569,589], Dafalias-Herrmann (boundary surface) [581], Tang (sand) [571], Fang [591], de Boer [505,600], Xing-Liu-Zheng (loess) [603], Yumlu and Ozbay [417], Wang-Ma-Zhou (dynamic characteristics of soil in complex stress state) [614], and others have indicated appreciable influences of the intermediate principal stress on the behavior involved in the stress-strain relations, pore pressure, and strength characteristics of most materials. It is obvious that the third stress (the intermediate principal stress) influences all three principal strains and the volumetric strain.

After many studies, Green [562] came to the following conclusion in the Roscoe Memorial Symposium held at Cambridge University in 1971: “Mohr-Coulomb failure criterion will tend to underestimate the strength by up to 5° of cohesive angle for the dense sand as the value of the intermediate principal stress increases. This would be a significant error in many analyses of engineering problems but represents an extreme case in as much as medium dense sand, loose sand, and probably most clays show a small increase.” Bishop [105,556] also indicated that the failure surfaces of extended Tresca and extended von Mises criteria are clearly impossible for a cohesionless material.

At the same Symposium, Harkness [108] indicated that: “The great disadvantage of the Mohr-Coulomb criterion at

present is the lack of indication of behavior in the direction of the intermediate principal stress. Further development of Mohr-Coulomb in this direction would be most interesting.” Some international symposia were held, the purposes of which were to allow a comparison to be made of various mathematical models of mechanical behavior of soils [339].

The introduction of a spherical end cap to the Drucker-Prager criterion was made by Drucker *et al* [550] to control the plastic volumetric change or dilation of soils under complex stress state. Since then, a specific Cam Clay model was suggested by Roscoe *et al* [553]. The Cam Clay model and critical state soil mechanics had been developed by the research group at Cambridge University [150,151,553,559,628,629]. The cap model had been further modified and refined by DiMaggio *et al* [631,632], Farque-Chang [637] *et al*. Critical state concept gained widespread recognition as a framework to the understanding of the behavior of soils, eg, Atkinson and Branoby [633], Atkinson [634], Wood [151], and Ortigao [638]. The critical state concept was applied also to rock by Gerogiannopoulos and Brown [386] and to concrete [31].

The cyclic behavior of soil under complex stress was studied by many researchers. A single-surface yield function with seven-parameter for geomaterials was proposed by Ehlers [626]. The multisurfaces theory was originally introduced by Mroz [639] and Iwan [640] and applied to two-surfaces or three-surfaces model by Krieg [641], Dafalias-Popov [478], Prevost [642,643], Mroz *et al* [568,576,635], Hashiguchi [147,616], Shen [644], Hirai [646], de Boer [505], Simo *et al* [647], Zheng [649] *et al*. A generalized nonassociative multisurface approach for granular materials was given by Pan [648]. The concept of the bounding surface was proposed by Dafalias and Popov [478] in metal plasticity, and applied to soil plasticity by Mroz, Norris, and Zienkiewicz [568,576,635], as well as Dafalias and Herrmann [581] and Borja *et al* [606].

Strength theory is also generalized to act as rigid-plastic and elasto-plastic models in RS (reinforced soils). Some criteria for RS were proposed, such as Sawicki [651], Michalowski and Zhao (RS with randomly distributed short fiber) [613]. A global yield surface considering the σ_1 and σ_3 was given by Sawicki. The summary of yield conditions for RS and its applications in RS structures was presented in Sawicki recently [618].

The joint failure criterion [437] and the Mohr-Coulomb failure criterion were adopted as the yield criterion of soil and interface in research for dynamic soil structure interaction system [595]. A new interface cap model was recently developed by Lourenco-Rots [650] that is bounded by a composite yield surface that includes tension, shear and compression failure as follows

$$F = c_{nn}\sigma_m^2 + c_n\sigma + c_{ss}\tau_s^2 = \sigma_0 \quad (66)$$

6.4 Failure criteria for iron

Studies of the fracture of iron date back to the work of Cook and Robertson in 1911 (thick-walled tubes subjected to internal pressure and compression) [207], Ros and Eichinger in

1926 (thin-walled tubes subjected to internal pressure plus tension) [289], and Siebel and Maier in 1933 [298]. Fracture and yield surfaces of iron have been studied also by Grassi-Cornet [652], Coffin-Schenectady [653], Fisher [654], Cornet-Grassi [655], Mair [657], Pisarenko-Lebedev [658], Yang and Dorztig [659], Hjelms [670], and others. Most results were obtained under bi-axial stress.

A modified Mohr-Coulomb criterion was proposed by Paul to fit the test data [18,108], and a modified von Mises criterion for iron was proposed recently by Hjelms in 1994 [670]. The comparisons of the twin-shear strength theory with the test data of Grassi-Cornet, Coffin-Schenectady, and Cornet-Grassi in the tension-compression region were given by Yu-He-Song [155]. It is shown that the agreements with experimental data are better than the Mohr-Coulomb theory. The maximum stress theory was used in the tension-tension region [10,12,18].

The yield surface for gray cast iron under bi-axial stress neither agrees with the Mohr-Coulomb theory nor the Drucker-Prager criterion [670]. A combined yield surface was formulated for gray iron by Frishmuth and Wiese as well as Yang and Dantzig [659].

6.5 Failure criteria for high strength steel and alloy

Yield criteria of metallic materials were further studied in the 1960s and 1970s [671–719].

The hydrostatic stress effect of metal materials were tested by Bridgman [224–228]. He reported the effects of hydrostatic stress on the fracture stress for a variety of alloys. The research works on high pressure were collected in his seven volumes of books, including the first paper presented in 1909 and the 199th paper presented in 1963. Recently, the effects of hydrostatic stress on mechanical behavior and deformation processing of materials were reviewed by Lewandowski and Lowhaphandu [324].

The strength difference at tension and compression (SD effect) of high strength steels, aluminum alloy, carbide alloy, etc, were observed in the 1970s [319–323]. Brittle fracture loci of tungsten carbide were studied by Takagi and Shaw [737]. The generalized failure criteria considering the SD effect and the influence of hydrostatic stress have to be used. All the yield criteria including the Tresca criterion, von Mises criterion, twin-shear yield criterion, and the unified yield criterion cannot be adopted for high strength alloy, which have the SD effect.

The yield surfaces of aluminum and magnesium, and application in automotive engineering, were discussed by Hilinski-Lewandowski-Wang [720] and Bryant [738]. Osaki and Iino [740] study stress corrosion cracking behaviors of high-strength aluminum alloys under a complex stress state.

6.6 Failure criteria for ice

A rational utilization of floating ice covers for various activities requires the knowledge of the strength of ice and the bearing capacity of ice covers. The surveys and studies were contributed by Hallam-Sanderson (UK), Maattanen (Finland), Schwarz (Germany), Scinha-Timco-Fraderkmy (Canada), and Sodhi-Cox (USA) in *Ice Mechanics* edited by

Chung [741]. Kerr [47,742] and Dempsey-Rajapakse [743] gave some reviews for the bearing capacity of ice and ice mechanics. As the indication of Kerr [47], there are as yet no reliable analytical methods to determine the bearing capacity of floating ice covers subjected to load. A major shortcoming of the published analyses for the bearing capacity of ice covers was a lack of a well-established failure criterion [47,742].

The failure criteria of ice were also studied by Szyszkowski and Glockner [744,745], Mahrenholtz, Palathingal, and Konig [746], ZP Chen and SH Chen [747], and others [748–759]. The size effect in penetration of sea ice was studied by Bazant-Kim [750] and Bazant and EP Chen [751].

The failure criteria of ice used for several decades were the well-known maximum normal stress criterion, maximum strain criterion, strain energy criterion [34,748,752] and others [746–756]. The maximum normal stress criterion, Mohr-Coulomb theory, and the twin-shear strength theory were used for ice by ZP Chen and SH Chen [747]. Recently, the choice of constitutive relations for a sea ice cover was discussed by Gol'dshtein and Marchenko [756]. The introduction of shear strength effects through a Mohr-Coulomb yield criterion plays an important role in determining ice drift in the marginal ice zone [757].

The compressive failure experiments of fresh-water ice under triaxial loading were given by Schulson and Gratz [758], and others. It is shown that the strength of the fresh-water is indistinguishable from that of porous salt-water ice. The failure process can be described by the Mohr-Coulomb criterion.

The research trend in ice mechanics was discussed by Dempsey [759]. It is much needed to find a reasonable failure criterion for ice.

6.7 Failure criteria for polymers

Polymers exhibit two types of failure: Yielding and crazing. The OSS (von Mises) criterion was sometimes used in polymers. However, many tests of polymers under a complex stress state show that the yield loci of polymers neither agree with the Tresca criterion or the von Mises criterion [32,760]. The yield and crazing criteria of polymers under complex stress were studied by many researchers [760–778].

Whitney and Andres [760] studied the behavior of polystyrene, polymethyl methacrylate, polycarbonate, and polyvinyl formulas under a complex stress state. The results did not fit either the Tresca or the von Mises criterion [760].

The effect of strength differences in tension and in compression and the effect of hydrostatic stress have to be considered for polymers. Bowder-Jukes [766] proposed two yield criteria for polymers in which the effect of hydrostatic stress was taken into account. This criterion is sometimes called the Bowder-Jukes criterion in polymer science. They can be expressed as follows

$$F = \tau_{13} + A\sigma_m = C; \quad F = \tau_8 + A\sigma_m = C \quad (67)$$

They are the generalized Tresca and generalized von Mises yield criteria. The maximum normal stress criterion, Tresca criterion, and von Mises criterion were generalized as damage surfaces for polymers by Tamuzs [772].

The unified strength theory, which will be described in the next section, was applied to one kind of polymer [156]. Yielding of polymers under complex stress was also investigated by Sternstein-Myers [765] and Giessen-Tvergarrd [773] in principal stress space.

The craze of polymers is different from the yield of polymers. However, craze zones of polymer structures under loading are similar to plastic zones of metallic materials [32]. Some craze criteria of polymers were proposed by Sternstein and Ongchin [761], Oxborough and Bowder [762], Raghava [763], Matsushige, Bear *et al* [764], Ward [32], and Argon, Hannoosh, and Salama [768–770], and others. Argon proposed a theory of crazing based on physical ideas, which introduces the influence of the deviatoric stress and hydrostatic stress as essential components of the initiation and growth mechanisms.

Sternstein and Myers [765] formulated that crazing occurs once the complex stress condition is satisfied

$$\sigma_1 - \sigma_2 \geq \frac{B}{\sigma_1 + \sigma_2} - A \quad (68)$$

where σ_1, σ_2 are the maximum and minimum principal stresses, respectively, and A and B are material constants. Kramer-Berger gave a review of craze growth and fracture in 1990. The experimental and theoretical studies, as well as the numerical simulation of craze, were given by Han, Giessen, Lai and others. A cohesive surface model for modeling crazing was proposed by Tijssens-Giessen-Sluis [778]. The concept of cohesive surfaces was used to represent crazes. The competition between shear yielding and crazing in glassy polymers was studied by Estevez, Tijssens, and Giessen [777]. Little data existed in the literature on the craze of polymers under complex stress state. The theoretical framework on initiation and breakdown of crazes is still not complete.

6.8 Failure criteria for energetic materials (TNT, RDX, solid rocket propellant, etc)

Energetic materials includes solid propellant, explosive materials (TNT-trinitrotoluene, RDX-cyclotrimethylen trinitramine and the Composition B-a composite of TNT and RDX, etc), and others. The triaxial strength has been studied. The conditions for failure are very important for the safe use of these materials.

Solid rocket propellant is a special material. Its mechanical behavior is similar to that of polymers. The strength of the propellant under complex stress was studied by Jones *et al* [779,780], Zak [781], Darwell, Parker, and Leeming [782], Sharma *et al* [783,784] and others [779–790]. A von Mises-Drucker-Prager type creep-damage model for solid propellant under complex stress was presented by Shen [787]. A bi-axial test facility for solid propellant was studied by Xie and Tang [788]. The tests of Kruse-Jones [780] and others showed that solid rocket propellant is pressure-sensitive, so a two-parameter failure criterion for propellant is needed. The constitutive models for propellants were investigated by Swanson-Christenson [785] and Finne-Futsaether-

Botnan [786]. Qiang [789,790] used a new strength theory called the unified strength theory (see next section) for propellant.

The triaxial yield properties of energetic materials (TNT and a composite of TNT and RDX) were given by Pinto and Weigand [791]. On the basis of experimental curves of energetic materials under the uniaxial and triaxial compression, a method of computer numerical modeling combining these curves was given by Zhang *et al* in ISSTAD '98 [792]. The experimental curves under the conditions of triaxial confined compression were modeled by using a finite element model with the Mohr-Coulomb friction contact element for the sample-steel cylinder system [792].

6.9 Failure criteria for ceramic, glass, etc

The effect of polyaxial stress on failure strength of ceramics was studied by Broutman-Cornish [793], Botdorf-Croze [694], Lamon [796], Sturmer-Schulz-Wittig [797], and others. The normal stress criterion, strain energy criterion, and other criteria were used. The investigations of Sturmer-Schulz-Wittig [797] indicate that the selection of the correct fracture criterion becomes even more important than for calculations based on fracture. The fundamentals of multiaxial failure criteria of ceramics and the experimental methods were described in Chapter 10, "Multiaxial failure criteria", of the book entitled *Ceramics: Mechanical Properties, Failure Behavior, Materials Selection* [54].

Failure criteria were used to study the hypervelocity penetration of tungsten alloy rods into a ceramic target in order to quantify the ballistic efficiency by Rosenberg *et al* [799].

Gurney and Rowe [801], Taylor [800], Davigenkov and Stabrokin [802], and Handin *et al* [375] studied the fracture of glass and similar materials. Richard [794] gave the limit loci of three graphites under plane stress.

The strength of a sintered aluminum ceramic under biaxial compression was determined by Adams and Sines [795]. Lade [608] gave a comparison of his test results with a general 3D failure criterion.

6.10 Failure criteria for other materials

Cellular material solid foams. In many applications, foams, including the rigid polymer foam, lightweight cellular concrete, metallic foams, ceramic foams, etc, are subjected to multiaxial stresses. Solid foams are macroscopic discontinuous materials. The multiaxial failure criteria are phenomenological, it is of importance for designers. Shaw and Sata [803] first measured the failure of foams under multiaxial stress. Their results indicated that under biaxial compression foams yield according to a maximum principal stress criterion. A systematic investigation regarding the multiaxial failure of foams was done by Ashby *et al* at Cambridge University and the Gibson group at MIT of USA [804–809,814,815]. Theocaris [810] proposed an elliptic parabolic failure criterion for cellular solids and foams. A failure criterion for tensile rupture of foams was written as follows:

$$F(I_1, J_2) = \pm \sqrt{J_2} - 0.2aI_1 = \sigma_{cr} \quad (69)$$

This equation is of the similar type to the Drucker-Prager criterion for soils. The limit surfaces in stress space consist of two intersecting surfaces of conical shape associated with the tensile and compressive limits (Triantafillou-Gibson [807]). Failure surfaces for cellular materials under multiaxial loads were given by Triantafillou *et al* [805,806].

The yield surfaces of aluminum alloy foams for a range of axisymmetric compressive stress states have been investigated by Deshpande and Fleck [808]. Two phenomenological isotropic models for plastic behavior were proposed. Good agreement is observed with the experimental results.

Aluminum foams are currently being considered for use in lightweight structural sandwich panels and in energy absorption devices. In both applications, they may be subjected to multiaxial loads. The designer requires a criterion to evaluate the combination of multiaxial loads which cause failure. Two phenomenological yield surfaces gave a good description of the multiaxial failure of the aluminum foams tested in the study of Gioux *et al* [815].

An experimental study of triaxial compressive dynamic mechanical properties of four different polyurethane rigid foam plastics at different temperatures and strain rates were done by Yin, H Li, and Han [813].

Smart materials: Piezoelectric solid, shape memory alloy. The plastic behavior of piezoelectric ceramic was first described by pressure sensitive transformation criterion by IW Chen and Reyes-Morel [816]. The criterion is expressed as follows:

$$\frac{\tau_8}{A} + \frac{\sigma_m}{B} = 1 \quad (70)$$

where A and B are material parameters.

A significant difference between tension strength and compression strength in shape memory alloys was observed in the experimental works. It has been found by Huang and Fleck [817] that the yield surface (phase transformation start stress) did not really match the von Mises criterion. A yield surface formula was given by Krenk [134] as follows:

$$(\sigma_1 - \sigma_m - c)(\sigma_2 - \sigma_m - c)(\sigma_3 - \sigma_m - c) = -\eta c^3 \quad (71)$$

$$c = \frac{2}{9} \frac{\sigma_c^3 - \sigma_t^3}{\sigma_c^2 - \sigma_t^2}, \quad \eta = \frac{(2\sigma_c - 3c)(\sigma_c + 3c)^2}{9c^3}$$

where c and η are material parameters, σ_t and σ_c are yield stresses under uniaxial tension and compression, respectively. The analytical results of Huang and Fleck [817] agreed well with this expression and experimental results.

Multi-axial phenomenological constitutive laws for ferroelectric ceramics were studied by Lynch [818]. A quadratic yield surface in an electric field and stress space was used to study the multi-axial electrical switching of a ferroelectric by JE Huber and Fleck [819].

Photoplastic materials. The yield loci of photoplastic materials were studied by Whitfield and Smith [820], Raghava *et al* [763], Argon and Bessonor [768] and others. The experimental results of polycarbonate, glassy polymer, and cellular showed the yield loci different. The experimental re-

sults of silver chloride (AgCl) did not fit either the Tresca or von Mises criteria, and were close to the Mohr-Coulomb strength theory [823]. Some reviews can be found in two books [821,824].

Biomaterials. The strength of bone was studied by Cowin [825]. No failure theory for bone has been validated at this time. Keyak and Rossi [827] examined nine stress and strain based failure theories, six of which could account for differences in tensile and compressive material strength, to predicate the strength of femoral. These failure theories include the σ_{\max} criterion, ε_{\max} criterion, τ_{\max} criterion, γ_{\max} criterion, the Mohr-Coulomb theory, Modified Mohr-Coulomb failure criterion, and the Hoffman criterion. The Tsai-Wu anisotropic failure criterion [21] was applied to bovine trabecular bone by Keaveney and Wachtel [826].

Pietruszczak, Inglis, and Pande [828] proposed a fracture criterion for bone tissue. The fracture criterion was expressed as a scalar-valued function of the stress tensor.

Powder. The yield of powder metals is significantly influenced by hydrostatic stress. Schwaiz and Holland [829] carried out a high-pressure triaxial test to establish the relationship between hydrostatic stress and shear stress for an iron powder. Shima and Mimura [830] performed a triaxial test of ceramic powder.

During the past three decades, several yield functions for porous materials including the P/M (powder metal) materials under complex stress have been developed [830–842], in which are included Kuhn and Downey [831], Shima and Oyan [832], Gurson [248,249], Tvergaard [250,251], Doraivelu *et al* [834], Kim and Suh [716], and Narayanasamy *et al* [842]. A combination of the Mohr-Coulomb criterion and elliptical cap model was applied to describe the constitutive model of powder materials by Khoei and Lewis [838]. A spheroidal yield surface in principal stress space and two other models were used for micro-mechanical modelling by Henderson *et al* [841].

Akisanya, Cocks, and Fleck obtained the shape of the yield surface of copper powder in 1997 [837]. The 1018 steel powder thin-walled tubular under torsion-tension (compression) test was done by Lade and Mazen [833]. Gothin *et al* [835] carried out an FE calculation employing a Mohr-Coulomb material model for the compaction of iron, bronze, ceramic, and carbon powders.

The failure loci are larger than that of the prediction of the Mohr-Coulomb theory. Park *et al* [839] proposed a new failure criterion for metal powder. The yield surfaces of compacted composite powder under triaxial test were measured and studied by Sridhar and Fleck [840].

Coating and adhesive etc. Micro-crack in the hard coating initiates usually from the local yield position. To prevent the crack from occurring, the most important criterion is to satisfy the condition that the equivalent stress of yield criterion is less than the yield strength of the material. The von Mises yield criterion was used to study the micro-crack initiation in the hard coating by Diao [843].

The Tresca and von Mises criteria were used to the limit

load solutions of adhesive by Alexandrov and Richmond [844]. Plastic yielding of a film adhesive under multiaxial stresses was studied recently by Wang-Chalkley [845]. It was found that the conventional yield criteria widely employed to model adhesives, such as the modified Tresca criterion, the modified von Mises criterion, and the linear Drucker-Prager criterion, are unable to characterize the yield locus. A modified Drucker-Prager cap model consisting of three yield surfaces was used to provide an adequate description of the yield locus for both tensile and compressive hydrostatic stresses. The design of a structural adhesively bonded joint is not completed by the lack of suitable failure criteria, as indicted by Sheppard *et al* [846]. Fatigue failure criterion of an adhesively bonded CFRP/metal joint under multiaxial stress conditions was studied by Ishii *et al* [847]. A damage zone model for the failure analysis of adhesively bonded joints was presented by Sheppard *et al* [846].

The experimental results of fatigue strength of a surface of a metallic material by Zhang, KW Xu, and JW He [849] showed the agreement with the twin-shear criterion.

The viscoelastic plastic analysis of lubricants was studied and summarized by YK Lee, Ghosh, and Winer [850].

Soft rock and coal. The plastic behavior of soft rock, including rock salt, potash, gypsum, etc, was usually described with constitutive models based on the elasto-plastic theory [403], creep condition, or internal state variables [851]. The true triaxial test and failure criteria were given by Chiu *et al* [397] and Hunsche [336,408,410] for rock salt. The OSS theory, ie, J_2 type theory or equivalent stress, was used as a yield or failure function in most cases. Aubertin-Ladanyi [851] proposed a function, which is similar to a viscoplastic yield criterion as follows:

$$F = \sqrt{J_2} - a_1(1 - \exp a_2 I_1) \cdot F(J_3) = C \quad (72)$$

Tests of the strength of coal under biaxial compression and triaxial compression were done by Hobbs [852,853]. The effect of intermediate principal stress was observed.

Recently, Medhurst and Brown [854] carried out a series of triaxial compression tests of coals. The Mohr-Coulomb criterion, modified Mohr-Coulomb criterion, and parabolic yield criterion were used to describe the viscoplastic constitutive model of rock-like materials and coal by Nawrocki and Mroz [421,424].

Brick masonry. The Mohr-Coulomb theory was often used for brick. A continuum model for assessing the ultimate failure of brick masonry as a homogenized material was given by Buhan and Felice [855] and others. The interface model was applied to fracture of masonry structures [856,857]. A three-parameter hyperbolic yield criterion was proposed for brick and masonry-infilled reinforced concrete frames by Mehrabi and Shing [546]. The failure criteria were also studied by Sabhash and Kishore [858] and by Sinha and Ng [859]. A multisurface interface model was used for masonry structures [650]. Recently, a review of state-of-the-art techniques for modelling masonry, brickwork, and blockwork structures was given in a special book [860].

Other materials. The strength of various materials under complex stress states were studied widely [861–882]. The relationship between shear strength and normal stress of municipal solid waste was tested by Eid *et al* in 2000 [861]. The results showed that the shear strength of solid waste increased with increasing normal stress. The Mohr-Coulomb strength theory was applied to study the stability of waste slope by Eid *et al* [861].

7 UNIFIED STRENGTH THEORIES

Experimental investigations have led to a substantial amount of knowledge regarding the strength of materials under the complex stress state, along with recent developments of numerical methods and computer application that have made possible the consideration of the use of a more refined or perfect strength theory. The theory is expected to have the following characteristics:

- 1) It should be able to reflect the fundamental characteristics of rock, concrete, and geomaterials, viz different tensile and compressive strengths, hydrostatic pressure effect, normal stress effect, and the σ_2 effect, and give good agreement with existent experimental data. The yield criteria for metallic materials are special cases of the expected strength theory.
- 2) It should be physically meaningful and should be expressed by mathematically simple equations to the maximum extent possible; It should have a unified mathematical model and a simple and explicit criterion which includes all independent stress components.
- 3) It should also be suitable for different types of materials under various stress states, but the minimization of the number of material parameters sufficiently representing material response, and incorporate various failure criteria from convex to non-convex, and encompass well-known failure criteria as special cases or linear approximation, and establish the relations among various failure criteria.
- 4) It should be easy (may be linear) for application to analytic solution and numerical solution.

All the yield and failure criteria mentioned above are the

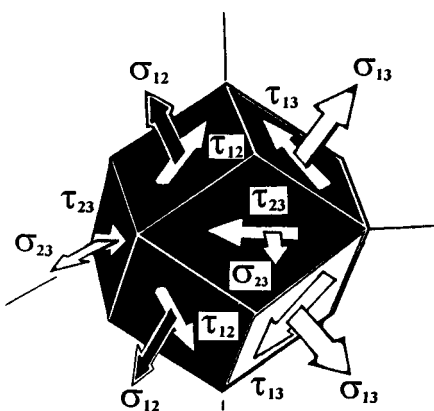


Fig. 8 Multi-shear model of the unified strength theory

single criteria adapted for one kind of material, respectively. We will introduce another kind of strength theories that can be adapted for more kinds of materials.

7.1 Octahedral-shear general strength theories

A united strength theory was proposed by Fridman [873,874] and Davigenkov [84]. This united strength theory was introduced widely in the USSR and in China before the 1970s. However, it is only a combination of the maximum shear stress criterion and the maximum strain criterion (or maximum principal stress criterion), as indicated in the *Encyclopedia of China* in 1985 [875].

Other general strength criteria are octahedral-shear typed theories or J_2 typed theories. This kind of generalized strength theory has been studied by DiMaggio and Sandler [631,632], Houlsby [592], Desai [39,579,864,876], Krenk [134], Shen [135,598] and Ehlers [626] in meridian sections for geomaterials. Desai [864] proposed a hierarchical single-surface model (HISS model). De Boer [600] proposed a function for soil. Shen [598] proposed a series model in the meridian section. Ehlers [626] proposed a seven parametric single-surface yield function for geomaterials. Valliappans presented a damage model as a unified strength theory [877]. Krenk [134] presented a family of limit surface considering the third invariant of deviatoric stress tensor. These models are able to describe the sensitivity of the plastic response of geomaterials to hydrostatic stress. They are the octahedral shear series of strength theories (or J_2 theory) described as follows:

$$F = J_2 + \alpha I_1^2 + \gamma I_1 + \beta I_1 J_3^{1/3} = K^2 \quad \text{or}$$

$$F = J_2 + (\alpha I_1^n - \gamma I_1^2) \left(1 - \beta \frac{J_3^{1/3}}{J_2^{1/2}} \right)^m$$

(Desai criterion) (73)

$$F = \left(J_2 + \frac{1}{2} \alpha^2 I^2 \right)^{1/2} (1 + \gamma \theta)^{1/3} + I_1 \beta = C$$

(de Boer criterion) (74)

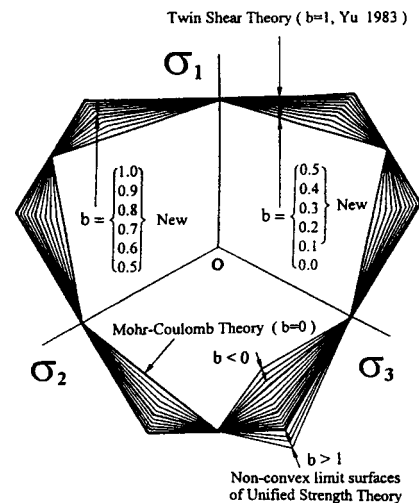


Fig. 9 Varieties of the unified strength theory on the deviatoric plane

$$F = \sqrt{J_2(1 + \gamma\theta)^m + \frac{1}{2}\alpha I_1^2 + \delta^2 I_4 + I_1\beta + I_1^2\epsilon} = C, \quad (\text{Ehlers criterion}) \quad (75)$$

$$F = J_3 + cJ_2 - (1 - \eta)c^3 = 0 \quad (\text{Krenk criterion}) \quad (76)$$

$$F = \frac{\sigma_m}{1 - (\eta/\eta_0)^n} \quad (\text{Shen criterion}) \quad (77)$$

where

$$\eta = \frac{1}{\sqrt{2}} \sqrt{\left(\frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2}\right)^2 + \left(\frac{\sigma_2 - \sigma_3}{\sigma_2 + \sigma_3}\right)^2 + \left(\frac{\sigma_3 - \sigma_1}{\sigma_3 + \sigma_1}\right)^2}$$

These failure functions contain a series of envelopes. The envelopes of Ehlers' yield function can be simplified to an open cone when the number of material parameters is reduced from seven to five. Two J_3 -modified Drucker-Prager yield criteria were proposed by Schreyer and Babcock [499,507]. Bardet proposed a Lode angle dependent failure criterion [509]. They are the octahedral shear typed criteria considering J_2 , I_1 , and J_3 . The forms of these failure criteria are similar as the OSS limit surfaces mediate between the SSS and TSS theories as shows in Fig. 2.

7.2 Twin-shear unified strength theory

A unified strength theory was proposed by Yu and He in 1991 [271,878], and further presented by Yu in 1992 and 1994. It can be found in Yu's book [56] and paper [879]. It was derived based on the concept of a multiple slip mechanism and the multi-shear element model shown in Fig. 8. The multi-shear element is a spatial equipartition available for continuum mechanics [879].

This element model is a rhombic dodecahedral multiple slip element differing from that of the principal stress cubic element used in common continuum mechanics. There are three sets of principal shear stresses and normal stresses acting on the same sections on which the principal shear stress are acting respectively.

$$\tau_{ij} = \frac{\sigma_i - \sigma_j}{2}, \quad \sigma_{ij} = \frac{\sigma_i + \sigma_j}{2}, \quad i, j = 1, 2, 3$$

There are only two independent components in three principal shear stresses, because the maximum shear stress τ_{13} equals the sum of the other two, ie, $\tau_{13} = \tau_{12} + \tau_{23}$. Considering the two larger principal shear stresses and the corresponding normal stress and their different effects on the failure of materials, a mathematical modelling of the unified strength theory can be formulated as follows [56,271,878,879]:

$$F = \tau_{13} + b\tau_{12} + \beta(\sigma_{13} + b\sigma_{12}) = C, \quad \text{when} \quad \tau_{12} + \beta\sigma_{12} \geq \tau_{23} + \beta\sigma_{23} \quad (78)$$

$$F' = \tau_{13} + b\tau_{23} + \beta(\sigma_{13} + b\sigma_{23}) = C, \quad \text{when} \quad \tau_{12} + \beta\sigma_{12} \leq \tau_{23} + \beta\sigma_{23} \quad (78')$$

where b is a coefficient reflecting the effect of the other principal shear stresses on the strength of materials. Introducing a tension-compression strength ratio $\alpha = \sigma_t/\sigma_c$ or $m = \sigma_c/\sigma_t$ the unified strength theory is expressed in terms of three principal stresses as follows:

$$F = \sigma_1 - \frac{\alpha}{1+b}(b\sigma_2 + \sigma_3) = \sigma_t, \quad \text{when} \quad \sigma_2 \leq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha} \quad (79)$$

$$F' = \frac{1}{1+b}(\sigma_1 + b\sigma_2) - \alpha\sigma_3 = \sigma_t, \quad \text{when}$$

$$\sigma_2 \geq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha} \quad (79')$$

or

$$F = m\sigma_1 - \frac{1}{1+b}(b\sigma_2 + \sigma_3) = \sigma_c, \quad \text{when}$$

$$\sigma_2 \leq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}$$

$$F' = \frac{m}{1+b}(\sigma_1 + b\sigma_2) - \sigma_3 = \sigma_c, \quad \text{when} \quad \sigma_2 \geq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}$$

The mathematical expression of this unified strength theory is simple and linear, but it has rich and varied contents, which can be easily changed to suit many new conditions. It possesses fundamentally all the above-expected characteristics. The limit surfaces of this unified strength theory in 3D principal stress space are usually a semi-infinite dodecahedral-sharp cone with unequal sides.

A series of limit loci of the unified strength theory on the deviatoric section are shown in Fig. 9 (and Fig. 2 when $\alpha = 1$). They are a dodecahedral locus when $b \neq 1$ or $b \neq 0$, or a hexagonal locus when $b = 0$ or $b = 1$.

As can be seen in Fig. 9, the unified strength theory is not a single criterion. It is a series of failure criteria, a system of strength theory. This theory gives a series of new failure criteria, establishes a relationship among various failure cri-

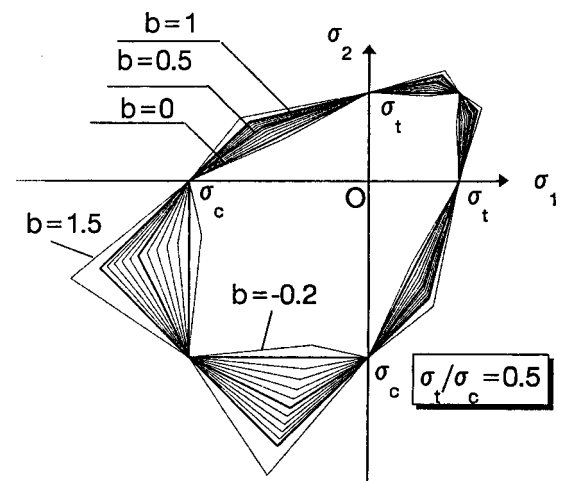


Fig. 10 Limit loci of the unified strength theory on plane stress

teria, and encompasses previous yield criteria, failure models, and other smooth criteria or empirical criteria as special cases or linear approximations. This unified strength theory has all of the desired characteristics mentioned above, and agrees with experimental results over a wide range of stress state for many materials including metal, rock, soil, concrete, and others. The unified strength theory can also be expressed in terms of stress invariant I_1 , J_2 , and J_3 . The detail descriptions can be found in Yu's paper [879] and books [55,56,156,273].

The unified strength theory can also be extended into various multiple parameter criteria for more complex conditions. The expressions are as follows:

$$F = \tau_{13} + b\tau_{12} + \beta_1(\sigma_{13} + b\sigma_{12}) + A_1\sigma_m + B_1\sigma_m^2 = C \quad (80)$$

$$F' = \tau_{13} + b\tau_{23} + \beta_2(\sigma_{13} + b\sigma_{23}) + A_2\sigma_m + B_2\sigma_m^2 = C \quad (80')$$

or

$$F = (\tau_{13} + \beta\sigma_{13})^2 + b(\tau_{12} + \beta\sigma_{12})^2 + A_1\sigma_m^2 = C \quad (81)$$

$$F' = (\tau_{13} + \beta\sigma_{13})^2 + b(\tau_{23} + \beta\sigma_{23})^2 + A_2\sigma_m^2 = C \quad (81')$$

These formulations are the non-linear unified strength theory. They can be used at the high-pressure stress region.

Equations (80) and (80') can be simplified to Eqs. (78) and (78'), when $A_1 = A_2 = 0$, $B_1 = B_2 = 0$ and $\beta_1 = \beta_2$. In this case, it is the single-shear strength theory (Mohr-Coulomb strength theory) when $b = 0$, or twin-shear strength theory when $b = 1$.

When $A_1 = A_2 = 0$, $B_1 = B_2 = 0$ and $\beta_1 = \beta_2 = 0$, Eqs. (80) and (80') are simplified to the unified yield criterion (60) and (60'), in this case, the twin-shear yield criterion and the single-shear yield criterion (Tresca criterion) are introduced when $b = 1$ and $b = 0$ respectively.

Equations (80), (80') (81), and (81') are nonlinear equations, which are not convenient for analytic solution in plasticity and engineering application.

7.3 Special cases of the unified strength theory

The unified strength theory contains four families of infinite criteria as follows:

- (a) Convex unified strength theory, when $0 \leq b \leq 1$;
- (b) Non-convex unified strength theory, when $b < 0$ or $b > 1$;
- (c) Convex unified yield criterion, when $\alpha = 1$ and $0 \leq b \leq 1$;
- (d) Non-convex unified yield criterion, when $\alpha = 1$ and $b < 0$ or $b > 1$.

The varieties of the unified yield criterion on the deviatoric section have been shown in Fig. 2. These yield loci can be adapted to all kinds of materials which have the same yield stress both in tension and in compression

The SSS theory (Mohr-Coulomb 1900) can be obtained from the unified strength theory when $b = 0$, ie,

$$F = F' = m\sigma_1 - \sigma_3 = \sigma_c \quad \text{or} \quad F = F' = \sigma_1 - \alpha\sigma_3 = \sigma_t$$

It is the lower bound of all convex limit surfaces. The formulation is the same as Eq. (4). It can be simplified to the Tresca yield criterion when $\alpha = 1$.

The TSS theory (Twin Shear Strength theory, Yu, 1985) can also be introduced from the unified strength theory when $b = 1$ as Eqs. (46) and (46')

A very simple, linear, and useful failure criterion is generated when $b = 1/2$. It is mediated between the SSS theory and the TSS theory. The expressions are as follows:

$$F = \sigma_1 - \frac{\alpha}{3}(\sigma_2 + 2\sigma_3) = \sigma_t, \quad \text{when} \quad \sigma_2 \leq \frac{\sigma_1 + \alpha\sigma_3}{1 + \alpha} \quad (82)$$

$$F' = \frac{1}{3}(2\sigma_1 + \sigma_2) - \alpha\sigma_3 = \sigma_t, \quad \text{when} \quad \sigma_2 \geq \frac{\sigma_1 + \alpha\sigma_3}{1 + \alpha} \quad (82')$$

The limit locus of this new criterion on the deviatoric plane is also shown in Fig. 9. For rock and concrete, most of the experimental failure envelopes fall in between the π -plane loci with $b = 1/2$ and $b = 1$ (Fig. 12). Therefore, the unified theory with $b = 1/2$ can serve as a new criterion, which can conveniently replace the smooth ridge models. The shape is

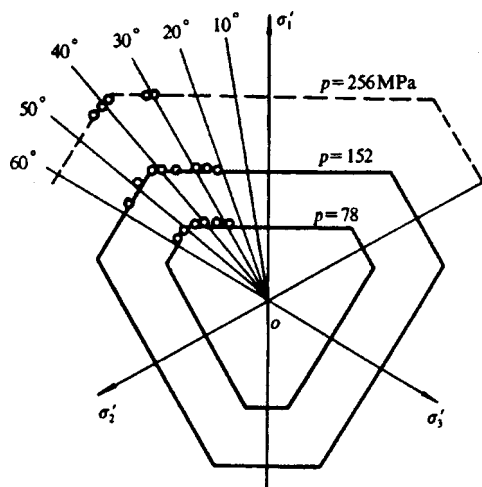


Fig. 11 Experimental results with the unified strength theory

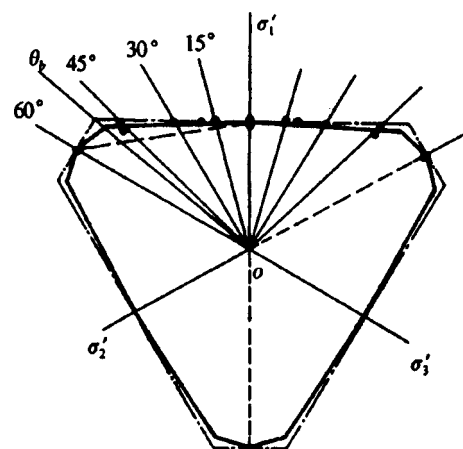


Fig. 12 Experimental results on sands with the unified strength theory

similar to the many empirical criteria and the numerically obtained limit surfaces from the other models. This new failure criterion may be a linear approximation of these criteria. This new failure criterion has been applied in the research on bearing capacity of a structure. When $\alpha = 1$ (ie, $\sigma_c = \sigma_t$), this criterion is simplified to

$$f = \sigma_1 - \frac{1}{3}(\sigma_2 + 2\sigma_3) = \sigma_t, \quad \text{when } \sigma_2 \leq \frac{1}{2}(\sigma_1 + \sigma_3) \quad (83)$$

$$f' = \frac{1}{3}(2\sigma_1 + \sigma_2) - \sigma_3 = \sigma_s, \quad \text{when } \sigma_2 \geq \frac{1}{2}(\sigma_1 + \sigma_3) \quad (83')$$

This new yield criterion is the approximation of the von Mises yield criterion. It may be referred to as a linear von Mises yield criterion or linear OSS criterion, and may also be a substitute for the von Mises criterion in an analytic solution to elasto-plastic problems [276–286].

In the biaxial stress state with $\sigma_3 = 0$, the shape of the limit loci of the unified strength theory is an asymmetrical dodecahedral locus when $b \neq 1$ and $b \neq 0$, or anti-symmetrical hexagonal locus when $b = 1$ and $b = 0$. Various failure criteria can be generated from the unified strength theory. The limit loci in plane stress state when $\alpha = 0.5$ are illustrated in Fig. 10.

It is emphasized that the ultimate justification of using a strength theory or failure criterion and its domain of validity depend on the ability of the resulting model to predict experimental data. The limit loci on the deviatoric section of the experimental results published in the literature are convex and lie in the range of $0 \leq b \leq 1$. Using the unified strength theory, it is easy to match various data.

The experimental results of three set specimens given by Michelis [98,99] are shown in Fig. 11. The solid lines are the limit loci of the twin-shear strength theory on the deviatoric plane at three different hydrostatic stresses. The comparison of the unified strength theory ($b = 3/4$) with the experimental results of Matsuoka and Nakai [577,601] is shown in Fig. 12. The comparisons of the unified strength theory with experimental results of about 28 materials presented in literatures relating the multiaxial strength of materials were given in [156].

The piecewise linear locus of the unified strength theory with $b = 1/2$ agrees with many data. The yield surface for gray cast iron under biaxial stress by Hjelm [670] is close to the unified strength theory with $b = 1/2$. The limiting loci of the unified theory fit quite closely with the corresponding test results on concrete by Launay and Gachon [163], Faruque and Chang [508], and others.

7.4 Applications of the unified strength theory

To summarize, the unified strength theory is a completely new system. It embraces many well-established criteria as its special or asymptotic cases, such as the Tresca, the von Mises, and the Mohr-Coulomb, as well as the twin-shear yield criterion [151], the twin-shear strength theory [155], and the unified yield criterion [271,272]. The unified strength

theory forms an entire spectrum of convex and non-convex criteria, which can be used to describe many kinds of engineering materials.

The unified strength theory is linear. It is convenient for application to the analytic solution of plasticity. This theory can also be expressed in terms of stress invariant [56,879] and it is convenient for computational implementation [880,887,892]. The singularity at the corners of the unified strength theory has been overcome by using a unified and simple method [880,887,892]. For more detailed discussions, interested readers are referred to the literature [880,887,892] and the books [56,55,156,273].

The theory has many connotations to be explored, and its study has been spreading and expanding quickly since 1997 [881–896]. Some unified solutions for plastic behavior of structures were introduced by using the unified strength theory [889–896]. The research results showed that the yield criterion has significant influence on the load-carrying capacities of plates. It was also indicated in these papers the exact results for metal materials obeying the linear unified yield criterion [283]. The unified strength theory has been applied successfully to analyze the dynamic response behavior for a circular plate under moderate impulsive load recently [286]. A series of analytical results were clearly illustrated to show the effects of yield criterion to elasto-plastic behavior [276–287], limit speed [279,282], and dynamic behavior [286,894,896]. Sometimes, the linear unified strength theory was referred as the Yu's unified strength theory [280,283–287,888,895,896]. The significance of the unification of the failure criteria and yield criteria was described by Yu, Zhao, and Guan [52] and Fan, Yu, and Yang [890].

Recently, a comment on the twin-shear strength theory and the unified strength theory was given by two Academicians of the Academy of China, Sun and SJ Wang [425], Senior Chairman and Chairman of the Chinese Society for Rock Mechanics and Engineering. Part of their statement is as follows:

“Constitutive laws of rocks provide the basis for the physico-mechanical simulation, numerical simulation, and computational analysis of rocks. They constitute the kernel problem for the theoretical study of rock mechanics. At present, they include elasto-plastic theory, rheology theory, and damage mechanics of rocks, etc. The macro-

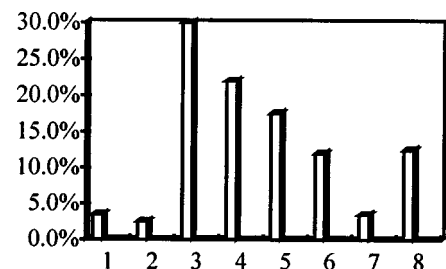


Fig. 13 Frequency of repetition of failure criteria: 1) Principal strain criterion, 2) Strain energy density criterion, 3) Maximum strain criterion, 4) Maximum stress criterion, 5) Tsai-Hill criterion, 6) Tsai-Wu criterion, 7) Strain ratio criterion, and 8) Others.

phenomenology has been developed and perfected with time in China. Representative work is as follows:

- 1) ...
- 2) ...
- 3) Maohong Yu (1985, 1990, 1997) proposed a theory of bi-shear strength and a unified theory of strength and postulated that yield surfaces in the space principal stresses can be expressed in the form of polyhedra which can be, in general, applied to metal, concrete, and rock materials. His rigorous study for years has continuously perfected the unified theory of strength, which has been applied to the design of underground projects and analysis of rock foundations in the realm of geotechnology" [425].

The unified strength theory can be generalized conveniently to more complex conditions as in Eqs. (80) or (81). This is the multi-parameters unified strength theory. All the unified strength theory (78), (78'), or (79), (79'), unified yield criterion (60), (60'), twin-shear strength criterion (48), (48'), twin-shear yield criterion (45), (45') and the single-shear strength criterion (Mohr-Coulomb theory), single-shear yield criterion (Tresca criterion) are special cases of this expression.

7.5 Extension: Non-convex strength theory

The ratio of shear strength to tensile strength of materials can be introduced from the unified strength theory as follows:

$$\alpha = \frac{\tau_0}{\sigma_t} = \frac{1+b}{1+b+\alpha}$$

It is shown that: 1) The ratio of shear strength to tensile strength $\alpha = \tau_0/\sigma_t$ of brittle materials ($\alpha < 1$) is higher than that of ductile materials ($\alpha = 1$), it agrees with the experimental data; 2) the limit surface may be non-convex when the ratio of shear strength to tensile strength $\alpha < 1/(1+\alpha)$ or $\alpha > 2/(2+\alpha)$; 3) the shear strength of a material is lower than the tensile strength of the same material. It is true for metallic material, it needs, however, to be further studied for other materials; 4) the unified strength theory has to be modified in the region of three tensile stresses states by adding a tension criterion or using the unified strength theory assumed α in Eq. (79). The unified strength theory with tension cutoff (similar to the Mohr-Coulomb theory with tension cutoff suggested by Paul in 1961 [107]) may be supplemented.

A series of non-convex failure surfaces can also be introduced from the unified strength theory when $b < 0$ or $b > 1$. The non-convex failure loci are shown in Fig. 3, Fig. 9, and Fig. 10. This kind of failure criterion has not been studied before, although non-convex limit surfaces based on experimental data are reported in some papers [275,551].

8 FAILURE CRITERIA FOR ANISOTROPIC AND COMPOSITE MATERIALS

Failure criterion for anisotropic and composite materials were studied by Hill [897], Hu and Marin [898–901], Smith [903], Goldenblat-Kopnov [904], Azzi and Tsai [906], Hsu [907], Bastun and Chernyak [909], Capurso [911], Lance and

Robinson [919], Shiratori and Ikegami [918], Tsai and EM Wu [21], Helfinstine and Lance [920], Lin, Salinas, and Ito [921], Chou-McNamee-Chou [924], Bastun [929], Dvorak-Rao-Tarn [926], and others [897–965].

Various anisotropic failure criteria and phenomenological failure criteria for composites were reviewed by Franklin [908], EM Wu [24], Tsai [933], Hosford [36], Rowlands [37], Budiansky [937], and Spottswood-Palazotto [953]. They are the maximum strain criterion, Petit-Waddoups criterion [913], maximum stress criterion, Hill criterion [897], Marin criterion [899], Norris criterion, Tsai-Hill criterion, Gol'denblat criterion [904], Ashkenazi criterion [905], Malmeister criterion [910], Hankinson criterion, Tsai-Wu criterion [21], Cowin criterion [825], Tennyson criterion [930], Hoffman criterion [912], Chamis criterion [914], Griffith-Baldwin criterion [902], Puppo-Evensen criterion [923], Dvorak-Rao-Tarn criterion [926–928], Hosford criterion [262], Hill's new criterion [268], tensor failure criteria [771], Voyiadis yield surface model [940], Hashin criterion [951], Ferron *et al* criterion [955], and some other criteria. Interested readers are referred to the literature [37,934,953]. Recently, a paraboloid invariant 3D failure criterion for transversely isotropic solids was proposed by Cazacu and Cristescu [945].

A user-friendly yield criterion was proposed by Hill [268], and used by Xu-Weinmann [943] and others. Five independent material parameters in presenting the yield locus were utilized.

A yield function for orthotropic sheet under plane stress condition [263], a six component yield function for anisotropic materials, and a new yield function for aluminum alloys [269,270,735] were established by Barlat and his co-workers [263,269,270,735]. Karafillis-Boyce [266] proposed a general anisotropic yield criterion using bounds and a transformation weighting tensor. The Hill's criterion is extended to a general orthotropic von Mises material model by Kojic-Grujovic and Zivkovic [950].

Soni [932] made an analysis of the frequency of repetitions regarding the use of various failure criteria for composites. The result of this investigation is shown in Fig. 13.

Various phenomenological descriptions of yielding and the effect of yield surface shape on prediction of forming limit and numerical simulation have been studied and developed by Ferron and his co-workers [955,956], Hopperstad *et al* [959], Frieman and Pan [960], and others.

A new six parameter general anisotropic yield surface using a fourth order anisotropic tensor was proposed by Voyiadis and Thiagarajan [940]. Lisenden-Arnold [941] gives the theoretical and experimental consideration of a flow-damage surface for metal matrix composites. An anisotropic yield criterion for polycrystalline metals using texture crystal symmetries was presented by Maniatty *et al* [947].

The unit cell analysis method [248,954] has been widely used in composite materials and mesomechanics [248,249,253,1000]. Micromechanical analysis of yield surfaces of a metal matrix composite by the method of cells was reviewed by Dvorak and Bahei [928], Aboudi [939], and

others. Some forms of unit cell were discussed [961]. Two IUTAM Proceedings relating to anisotropic solid were given [964,965].

9 MULTIAXIAL FATIGUE, CREEP, DAMAGE AND RELATED PHENOMENA

Multiaxial fatigue problems are rather recent research topics. They have been developing strongly since the beginning of the 1980s. Four international conferences dealing with this subject were held in the USA, UK, Germany, and France. Four proceedings edited by Miller and Brown [966], Brown and Miller [967], Kussmaul, McDiarmid, and Socie [968], and Pineau, Gailletaud, and Lindley [969] were published in the USA and UK. A large number of studies have now been devoted to this topic including: Test facilities and experimental techniques; Theoretical aspects and constitutive modeling; Finite element calculations; Low cycle fatigue; Cycle deformation and damage; Life-time prediction; Incipient cracking and crack growth; Out-of-phase and non-proportional loading; High temperature and transient loading; Creep-fatigue etc.

There is considerable overlap between the subjects, which illustrates the inherent cross-linking of the many facets of multiaxial fatigue. It had been reviewed by Krempl [23], Garud [983], and recently by You and SB Lee [49], and Gao and Brown [48], as well as Zhang-Akid [973], and Kim-Park [974]. The following strength theories or criteria of multiaxial fatigue were used.

- (a) von Mises stress criterion or von Mises strain criterion;
- (b) Modified von Mises approach (von Mises stress hydrostatic stress correction);
- (c) Tresca criterion;
- (d) Rankine approach;
- (e) Guest criterion, Gough criterion, Findley criterion [975], Rotvel criterion, McDiarmid criterion for high-cycle fatigue;
- (f) Strain plane approach (Brown-Miller criterion);
- (g) Strain energy density (Ellyin [979]);
- (h) Modified strain plane approach (Lohr-Ellison criterion);
- (i) Kandil-Miller-Brown criterion;
- (j) Dang Van *et al*;
- (k) Sum of energy density (elastic and plastic; normal and shear strain).

These criteria were found to be suitable for only one kind of material, respectively. In these criteria mentioned above, (a)–(e) are used for high cycle fatigue and (f)–(h) are used for low cycle fatigue. The criteria (e) and (f)–(h) are two-parameter criterion. The two-parameter or more than two-parameter criterion is comprehensively adopted for multiaxial fatigue as indicated by Gao and Brown [48]. The mathematical expressions of two of these criteria were proposed respectively by Lohr and Ellison [981] and Macha [970] are expressed as follows:

$$\alpha \Delta \gamma_{13} + k \Delta \varepsilon_{13} = C \quad (84)$$

$$\alpha \Delta \gamma_{13} + k \Delta \varepsilon_{13} = C \quad (85)$$

where $\Delta \gamma_{13}$ is the maximum shear strain on the fracture plane and $\Delta \varepsilon_{13}$ is the maximum normal strain on the fracture plane, a and k are constants used to select a particular form of criterion. If $a=0$, $k=1$, the maximum normal strain criterion results from the Macha criterion; if $a=1$, $k=0$, it is the maximum shear strain criterion.

The new fatigue criterion for multiaxial stress is presented on the assumption that an expected fatigue fracture plane is a result of the occurrence of the values and directions of principal strains. The fracture plane is determined by the maximum value of a linear combination of shear and normal strains in this plane. It is evident that the new multiaxial fatigue criterion is the generalization of the single shear stress criterion of Mohr-Coulomb. It can be referred to as the single shear series strength theory. Chu, Conle, and Bonnen [989] proposed a sum of energy densities (elastic and plastic) of normal and shear strain in the critical plane for high and low cycle fatigue. A gradient dependent multiaxial high-cycle fatigue criterion of the stress invariant was formulated by Papadopoulos [992]. A new multiaxial fatigue criterion for hard metals was proposed by Andrea *et al* recently [996].

A new multiaxial fatigue criterion may be presented on the basis of the twin-shear strength theory [155] as follows:

$$F = \Delta \gamma_{13} + \Delta \gamma_{12} + k(\Delta \varepsilon_{13} + \Delta \varepsilon_{12}) = C \quad \text{when } F > F' \quad (86)$$

$$F' = \Delta \gamma_{13} + \Delta \gamma_{23} + k(\Delta \varepsilon_{13} + \Delta \varepsilon_{23}) = C \quad \text{when } F < F' \quad (86')$$

The unified multiaxial fatigue criterion, which is the generalization of the unified strength theory [271], may be proposed as follows:

$$F = \Delta \gamma_{13} + b \Delta \gamma_{12} + k(\Delta \varepsilon_{13} + b \Delta \varepsilon_{12}) = C \quad \text{when } F > F' \quad (87)$$

$$F' = \Delta \gamma_{13} + b \Delta \gamma_{23} + k(\Delta \varepsilon_{13} + b \Delta \varepsilon_{23}) = C \quad \text{when } F < F' \quad (87')$$

The effect of hydrostatic stress (mean stress) and the stress triaxiality are taken into account in the unified multiaxial fatigue criterion. It is a very systematic fatigue criterion. A series of multiaxial fatigue criteria can be obtained from this new unified multiaxial fatigue criterion. The special cases of the unified fatigue criterion are as follows:

- (a) It is the Tresca fatigue criterion (single shear criterion) when $\alpha = 1$, $b = 0$;
- (b) It is the Mohr-Coulomb fatigue criterion when $b = 0$, it is the same as the Mohr's circle method;
- (c) It is twin shear fatigue criterion when $\alpha = b = 1$;
- (d) It is generalized twin shear fatigue criterion when $b = 1$;
- (e) It is linear approximation of the von Mises fatigue criterion when $b = 1/2$, and $\alpha = 1$. It is the same as the second invariant of deviatoric stress J_2 or shear energy viewpoints;
- (f) It is a series of two-parameter criterion when $0 \leq b \leq 1$;

- (g) It is a series of single parameter criterion when $\alpha = 1$, and $0 \leq b \leq 1$.

The X-ray stress measurements of residual stress relaxation in biaxial stress showed that the twin shear criterion agrees well with the experimental data and is better than the von Mises criterion [849]. The fatigue testing results of Sanetra and Zenner [988] showed the lifetime diagram of 30CrNiMo8 for bending, torsion, and combined loading follow an elliptical curve very exactly. The ratio of torsion stress amplitude and bending stress amplitude are 0.66 for four curves. It is in agreement with the twin shear fatigue criterion.

Strength theories are also used in the research on multi-axial creep, damage mechanics, and mesomechanics. Bailey [260] may be the first researcher in multi-axial creep in 1935. He suggested a function that is referred as the Bailey law in the theory of creep. It is a von Mises type function. Taira *et al* [20] indicated that the failure and rupture time depends on the failure criterion. 32 results of different researchers regarding the multi-axial creep were summarized in Table 7.6 of their book [20]. The von Mises type criterion was used in the above results. However, he showed that the experimental data do not agree with the von Mises criterion. The expression combining the octahedral shear stress τ_8 , hydrostatic stress σ_m , and maximum principal stress σ_1 was proposed by Hayhurst [997] as follows:

$$G(\sigma_{ij}) = \{\alpha\sigma_1 + \beta\sigma_m + (1 - \alpha - \beta)\tau_8\}^{-m} \quad (88)$$

Series studies on multi-axial creep rupture were done by Hayhurst *et al* [997–999], Henderson [1000], Hurst [723], and others. The elaborations of an appropriate rupture criterion require further tests and analyses. The establishment of appropriate failure criterion might best be achieved through a torsion test as advised by Henderson [1000] and Hurst [723]. A two-parameter criterion was used to model the multi-axial creep rupture by Othman and Hayhurst [999]. A σ_1 - τ_8 type function is

$$G(\sigma_{ij})[\sigma_1^\alpha \cdot \tau_8^\beta]^{-m} \quad (89)$$

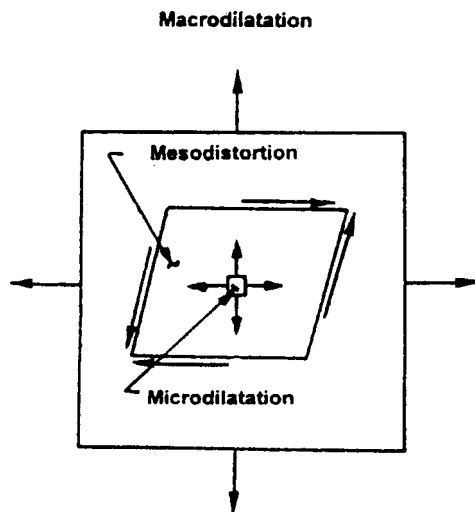


Fig. 14 Material element in Macro-Meso-Micro scale level(s)

Goncalves Filho [66] presented two 3D FEM solutions to the creep-rupture problem of a cruciform specimen under equal triaxial tension in detail. The tri-axiality of damage is often expressed by a τ_8 type combined function as follows:

$$\sigma^* = \alpha\sigma_1 + \beta\sigma_m + \gamma\tau_8 \quad (90)$$

Most of these researches on damage use the von Mises (OSS) typed criterion for metallic materials, and the Mohr-Coulomb's single-shear (SSS) criterion for geomaterials. The maximum tensile stress criterion was assumed by Kachanov, and the Tresca criterion was assumed by Rabotnov in 1958 and 1966, respectively, in damage and creep problems. Damage models for concrete were studied [506,517,529]. A Drucker-Prager type creep damage model of solid propellant was proposed by Shen [787]. A new plastic-damage model was proposed and used to analyze reinforced concrete plate by Wang and Fan [1013]. This new damage criterion is a combination of the unified strength theory (Yu) [271,273] and the experimental results on the strength of concrete by Kotsovos [481].

Li-Zhang used the twin shear failure criterion also as a damage function in concrete compression. It shows that the result is better than the Mohr-Coulomb theory when compared with tests [156]. GP Li and Tao proposed a micro-mechanical damage model for rocks subjected to true triaxial stresses [415].

An approximate yield criterion for a voided material based on a unit cell analysis was first derived by Gurson [248,253] as follows:

$$\Phi(\sigma_e, \sigma_m) = \left(\frac{\sigma_e}{\bar{\sigma}}\right) + 2fq_1 \cosh\left(\frac{\sigma_m}{2\bar{\sigma}}\right) - (1 + q_1^2 f) = 0 \quad (91)$$

where $\sigma_e = (3s_{ij}s_{ij}/2)^{1/2}$ is the effect stress and s_{ij} is the stress deviator, $\bar{\sigma}$ is the flow stress, and f is the void volume fraction. The Gurson model for porous ductile metals is based on an approximate limit-analysis for hollow spheres made of rigid ideal-plastic material using the von Mises yield criterion. Some modified Gurson models incorporating the influence of void shape and the effect of strong gradients of macroscopic fields were proposed by Tvergaard [250–252], Gologanu *et al* [253,712], and others.

A new constitutive model for rate dependent plasticity of porous solids was presented by Fotiu and F Ziegler [869]. It is shown that in plane stress and plane strain the static yield surface closely fits Gurson's yield surface and is similar to Tvergaard's [250] modified Gurson model in introducing proportionality. The yield functions can also attain a form similar to that of Mear and Hutchinson [1153]. The dynamic yield surfaces and yield loci in plane stress and plane strain and their applications have been summarized and critically reviewed by F Ziegler [1002,1003,1052] and Izschi and F Ziegler [1053].

An approximate dynamic yield criterion was introduced by Wang-Jiang [1057] for porous ductile media obeying the von Mises yield criterion.

The von Mises criterion was also used in plastic-damage theory. The von Mises criterion, however, is suitable only for

those materials whose yield stress is the same both in tension and in compression, and the ratio of shear yield stress with tensile yield stress equals 0.578. The damage models were studied by Lemaitre-Chaboche, Rousselier, and Hansen-Schreyer, Neilsen-Schreyer, Voyiadjis-Kattan, Yazdani-Karnawat (see, Ju [1006]), and others. Yield criteria and failure criteria were used also for the researches on shear band, discontinuum bifurcation, damage, impact, mesomechanics, etc [1001–1057].

The maximum normal criterion, the Tresca, and von Mises criteria were used to study the fracture micromechanics of polymer materials by Tamuzs [772]. A detail literature before 1981 can be found in the book of Kuksenko and Tamuzs [1049].

The stress triaxiality functions σ_e/σ_m or σ_m/σ_e are often used in fracture mechanics and damage mechanics. Obviously, it is similar to the Drucker-Prager criterion. Other forms of the stress triaxiality function may be used in the future. Unit cell analysis has been generalized and widely used in composite materials and mesomechanics. The computation modelling of materials failure was reviewed by Needleman [1035].

Shear band problems (strain localization) have been discussed in connection with strain softening, localization, discontinuous bifurcation, characteristics, and material instability. These problems were studied by Hill [291,305], Prager [297], Thomas [295,309], Rudnicki-Rice [1016], Rice [1017], Asaro-Rice [1018], Needleman [1019], Bai [1020], Peirce-Asaro-Vardoulakis [1021], Tvergaard [250–252,1031], Fleck-Hutchinson-Tvergaard, Bazant *et al* [1023], GC Li and Jaener [1025], Ottosen and Runesson [1030], Aifantis [1036], Zhang and Yu [1060], and others. Some reviews of papers on material instabilities and computation modelling were given by Zbib and Aifantis [710], Tomita [1042], Needleman [1035,1095], and others. In these researches, the von Mises, the Drucker-Prager, the Rankine, the Mohr-Coulomb criteria, and the critical stress criterion and critical strain criterion were used.

The unified strength theory is extended and applied to the researches of discontinuous bifurcation and concrete under high-speed penetration [535,894]. Micromechanical modelling of yield loci were studied by Lin *et al* [313,318], Zheng-Wei [532], Buyukozturk [541], and Wellerdick-Wojtasik [948]. It is shown that macroscopic properties can be obtained from an averaging procedure of micromechanical modelling. The shape of the calculating yield loci differs only slightly from the ellipsoidal shape of the von Mises locus [948]. The material models in mesomechanics and macromechanics were briefly discussed [961].

The stress state of a material element may be changed at the different scale levels [1162] as shown in Fig. 14. All the material elements, however, at the different scale levels are acted upon under the complex stress state. Gradient effects at macro, micro, and nano scales were researched by Aifantis *et al* [1036]. Strength theory is also studied and used in mesomechanics. The questions are: What combination of stress will cause the yield and failure? What are the same, and what are different at micro, meso, and macro scales? It is a very

interesting time for research on strength theory in the 21st century. The research trends of solid mechanics and strength theory were discussed in [1148] and [1161].

10 COMPUTATIONAL IMPLEMENTATION

Strength theory (yield and failure criterion, or material model), as the one of the most important constitutive relations, has been implemented into various computational codes, especially the non-linear computer codes based on the Finite Element Method (FEM). The earliest applications of FEM to plasticity problems are attributed to Gallagher-Padlog-Bijland [1058], Argyris [1059], Pope [1060], Reyes-Deere [1062], Marcal-King [1063], Yamada-Yashimura-Sakurai [1064], Zienkiewicz-Valliappan-King [1065], Richard-Blacklock [1066], and Pifko *et al* [1067]. Further papers and books were written or edited by Oden [1068], Nayak and Zienkiewicz [1069], Argyris *et al* [1071], Desai [39,399,1040], Gudehus [1072], Lippmann [1121], Owen-Hinton [1073], Desai *et al* [1040,1106,1134], Owen-Hinton-Onate [1098], Doltsinis [1105], Bangash [543], Kobayashi [1108], and others [1058–1119].

The yield criteria have also been implemented into the Boundary Element Method (BEM) codes (Telles and Brebbia [1074], Brebbia [1079], and others). The result of investigations of the relationships between yield criteria and press performance (formability) shows that FEM simulations of sheet forming operations depend strongly on the choice of the yield surface shape [958].

In general, these material models are the Tresca-Mohr-Coulomb single-shear series (SSS) and the von Mises-Drucker-Prager octahedral shear (OSS) series of strength theories. A reference book on the topic is available [258].

The form of yield surfaces of the single-shear series of strength theories is angular in the π -plane, the flow vector is not uniquely defined at the corners of the Tresca and Mohr-Coulomb criteria, and the direction of plastic straining there is indeterminate. Koiter [292] has provided limits within which the incremental plastic strain vector must lie. These singularities give rise to constitutive models that are difficult to implement numerically. To avoid such singularity, Drucker and Prager [117] have introduced an indented von Mises criterion in which the ridge corners have been rounded. The Drucker-Prager criterion has been widely implemented into nonlinear FEM codes and widely used for geomechanics and in geotechnical engineering. Unfortunately, this gives a very poor approximation to the real failure conditions as indicated by Humpheson-Naylor [118], Zienkiewicz-Pande [119], WF Chen [31], and WF Chen and Baladi [33]. It is owing to the fact that circular limiting loci in the deviatoric plane of the Drucker-Prager criterion contradicts experiments for geomaterials. Therefore, a lot of smooth ridge models were proposed by Gudehus [127,128], Argyris-Faust-Szimmat-Warnke-Willam [126], Willam-Warnke [120], Lade-Duncan [122], Matsuoka-Nakai [121,577], Dafalias [478,578], Lin and Bazant [129], Podgorski [132], Jiang [44,140], Guo-Wang [137,138], Menetrey-Willam [133], Song-Zhao-Peng [141,142,518], and others. Most of them are of the

octahedral-shear type (ie, J_2 theory) function expressed as in Eqs. (12)–(44). Various forms can be summarized into the expression as follows:

$$F = f(J_2) + f(I_1) + f(J_3) = 0,$$

$$F = f(J_2) - f(I_1)f(J_3) = 0 \quad (92)$$

or

$$F = f(\tau_8) + f(\sigma_8) + f(\theta) = C,$$

$$F = f(\tau_8) + f(\sigma_8)f(\theta) = 0 \quad (93)$$

At the same time, the singularities of the Tresca and Mohr-Coulomb yield criteria were also overcome by rounding off the corners of the surface or employing a simple mathematical artifice in the numerical procedure [1073]. The accurate treatments of corners in yield surfaces were studied by Marques [1090], Ortiz-Popov [1092], Sloan-Booker [1089], de Borst [1094,1099], Yin and Zhou [1091,1093], Runesson, Sture *et al* [1096], Simo-Kennedy-Govindjee [647], Pankaj-Bicanic [1097], Khan-Huang [866], Larsson-Runesson [1110], Jeremic-Sture [1111], Foguet-Huerta [1111], and others. So, the single shear type yield criteria are easy to use and easily implemented into computational codes. Recently, the singularity of the Tresca plasticity at finite strains was studied by Peric and de Neto [1112].

The yield criteria have been implemented into the most current commercial FEM systems, such as ABAQUS, ADINA, ANSYS, ASKA, ELFEN (U of Wales Swansea), MSC-NASTRAN, MARC, NonSAP, AutDYN, DYNA, DYPLAS (Dynamic Plasticity), etc. In some system, only von Mises and Drucker-Prager criteria were implemented. The functions and the applied field of many powerful commercial FEM codes were limited to the choosing of failure criteria. More effective and systematical models of materials under complex stress are demanded.

Recently, a new and effective 3D finite difference computer program, FLAC-3D (Fast Lagrangian Analysis of Continua in 3-Dimensions), is presented [1117]. The stability analysis on the high slopes of Three-Gorges shiplock using FLAC-3D was given by Kou-Zhou-Yang [1118]. It is a pity, however, that only one failure criterion-Drucker-Prager criterion was implemented into this code. As indicated by Humpheson-Naylor [118], Zienkiewicz-Pande [119], and WF Chen [31], and others, it is basically a shortcoming of the Drucker-Prager surface in connection with soil-strength modelling: the independence of τ_8 on the angle of similarity θ . It is known that the trace of the failure surface on deviatoric planes is not circular [31,33].

The twin shear strength theory has been implemented into special finite element programs by An-Yu [181], Yu-Meng [604], and others. The singularity has been overcome. It is easy to use. The twin-shear yield criterion and the twin-shear strength theory have been implemented into three commercial FEM codes by Quint Co. [182–184].

The unified yield criterion and the unified strength theory have been implemented and applied to some plasticity and engineering problems, eg, Yu-He-Zeng [274], Yu-Zeng [880], Yu-Yang-Fan [887,892], and others. The singularities

at the corners of single-shear series of strength theory, twin-shear series of strength theory, and the unified strength theory have been overcome by using a unified numerical procedure (UEPP Code [156,897,892]).

As use of FEM and other numerical analyses expand in engineering design with increased access to computers, it becomes important that strength theory (yield criterion, failure criterion) relating stress be carefully chosen. In adopting a criterion for use it is important that at least as much concern be directed to the physics of the problem and to the limitation of criteria. When it becomes necessary to adopt a criterion for use, it is important to experimentally check the criterion, or to investigate the experimental data in literature. If this is not done, then very exact numerical procedures or commercial codes can lead to completely worthless results. The shape of the yield surface is found to have a significant effect on the local deformations predicted in the simulations [959].

A Constitutive Driver, ie, a computer program containing a library of models where the tests can be simulated on the constitutive level and where parameter optimization can be performed, for soil plasticity models has been proposed by Mattsson, Axelsson, and Klisinski [621]. Four soil models have, so far, been included in the Constitutive Driver. The FEM was also used to study triaxial specimens by Calloche and Marquis in 1996 [344].

11 INTERNATIONAL CONFERENCES ON STRENGTH OF MATERIALS AND STRUCTURES UNDER COMPLEX STRESS

A series of IUTAM (International Union of Theoretical and Applied Mechanics) symposia on the strength of materials and structures was held during the last three decades. These proceedings were edited by Hult [1120], Lippmann [1121], Tryde [753], Nemat-Nasser [1122], Proter and Hayhurst [1123], Vermeer and Luger [1124], Bazant [1125], Bodner and Hashin [1001], Boehler [964], Dvorak [965], Zyczkowski [1126], Ortiz and Shih [1127], Baker and Karihaloo [798], Carpinteri [1128], Pineau and Zaoui [1004], Falachier, Lumley, and Auselmet [1131], Fleck and Cocks [1129], Bruhns and Stein [1130], Ehlers [1132], and others. Most materials in structures are subjected to complex stress states, ie, biaxial and multiaxial stresses. Strength theory provides a yield (or failure) criterion, that is, a limiting stress state for elasticity, or an initial deformation of plasticity. Sometimes, it is also used as an associated or non-associated flow rule for plastic constitutive relations.

Various conferences on constitutive relations of materials were held during the last two decades [1120–1148], including the International Workshop of Constitutive Equations for Granular non-Cohesive Soils edited by Saada and Bianchini [1135], International Conference on Constitutive Equations, Macro and Computational Aspects (Willam [1136]), Constitutive Laws and Microstructures (Axelrad and Muschik [1137]), Constitutive Laws in Engineering Materials (Desai *et al* [1040,1106,1134]), Constitutive Laws of Plastic Deformations and Fractures (Kransz [1065,1080]), International Symposium on Constitutive Laws held in conjunction with

the International Conference on Engineering Science (Rajendran [1138]), Constitutive Modelling of the Large Strain Behavior of Rubbers and Amorphous Glassy Polymers (PD Wu and Giessen, 1994 [776]), Constitutive Modelling of Granular Materials (Kolymbas [1140]), Constitutive Models of Deformation (Chandra and Srivastav, 1987 [1139]), Constitutive Relations for Soils [588], etc. The yield criteria for metals, concrete and soils were summarized by WF Chen in his two volume book entitled *Constitutive Equations for Engineering Materials* [41,42].

A series of Proceedings of International Symposia on Numerical Models in Geomechanics (NUMOG) [1141–1145] were published since 1982 [1141–1143]. Strength theories including yield and failure criteria of materials under complex stress were studied and used by many researchers in the constitutive equations (laws, relations, modelling, models), for plasticity, damage, and fatigue. Strength theories were also widely studied and used at other international conferences, such as: “Computer Methods and Advances in Geomechanics,” “Modelling and Computers in Geomechanics,” “Numerical Methods in Geomechanics,” “Continuum Models of discrete Systems,” and a series of “International Symposium on Plasticity and Its Current Applications” organized by Khan since 1981, and so on. The von Mises criterion, Drucker-Prager criterion, and the Mohr-Coulomb theory were widely used in the research on localization of plastic deformation in the *Proceedings of Plasticity '91* [1146]. The latest 8th Symposium on Plasticity 2000, entitled *Deformation of New Engineering Materials under Multi-Axial Conditions*, has been held in Japan.

Some special conferences on multiaxial strength of materials were held, such as: International Conference on Concrete under Multiaxial Conditions (Toulouse, France, 1984), Multiaxial Plasticity, and a series of International Conferences on Biaxial/Multiaxial Fatigue. The *First Proceedings of the International Conference on Biaxial/Multiaxial Fatigue* was published in 1985 [966]. The following five proceedings were published, edited by Brown and Miller [967], Kussmanl *et al* [968], Pirean *et al* [969], and Macha *et al* [974]. The book, entitled *Multiaxial Fatigue* [971], was published recently. The proceedings of the CNRS international colloquium on *Failure Criteria of Structured Media* was edited by Boehler [1147].

The International Symposium on Strength Theory: Application, Development, and Prospects for the 21st Century (IS-STAD '98) was held in Xi'an, China in 1998. The Symposium was co-organized by Nanyang Technological University, Singapore, Xi'an Jiaotong University, University of Hong Kong, and Tsinghua University, China. The symposium was also co-sponsored by the International Association for Computer Methods and Advances in Geomechanics (IACMAG). Nine keynote papers were given by Ansari [1149], WF Chen [51], Gong [1150], Sano *et al* [836], Shen [1148], Sih [1151], Valliappan [877], Voyiadjis *et al* [1012], and Yu [1152]. Another 177 papers relating the strength theories and their applications were included in the Proceedings [1148].

The Symposium demonstrated a great variety of recent

developments, implementations, applications, and verifications of a spectrum of strength theories of engineering materials, ranging from the simplest to the unified and sophisticated ones, which covered both simple and complex stress (multi-axial stress) states of many common engineering materials (such as metallic materials, rock, soil, concrete, and composite materials). The papers contributed to the progresses of strength theories in many major areas including static, dynamic, impact, and cycle strength properties; fracture, damage, fatigue, and creep investigations; applications in numerical modelling; engineering application, and experimental techniques.

12 CONCLUDING REMARKS

The complex stress state exists widely in nature and engineering. Strength of materials and structures under the complex stress state is a general problem. Strength theory is an important foundation for research on the strength of materials and structures, and is used widely in mechanics, physics, material science, and engineering. It is of great significance in theoretical research and engineering application, and is also very important for the effective utilization of materials. Hundreds of models (criteria) have been described in the 20th century, ranging from the one-parameter model (criterion) to the multi-parameter models.

Most of them are the single strength theory adapted for only one kind of material. No relationship exists among these criteria. These criteria, however, can be categorized into three series of strength theories. They are the series of Single-Shear Strength Theory (SSS Theory), the series of Octahedral Shear Strength Theory (OSS Theory), and the series of Twin-Shear Strength Theory (TSS Theory). The summaries of these three series of strength theories were given by Yu [29,53,426], Shen [46,52], and recently by Yu [156].

The SSS theory (Tresca-Guest-Mohr-Coulomb-Hoek-Brown *et al*) forms the lower (inner) bound for the entire possible convex limit surfaces on the π -plane. The OSS theory is a nonlinear function; it forms curved limit surfaces mediated between the SSS theory and the TSS theory. The TSS theory (Twin-Shear Strength Theory) is a new series of strength theory. It is also a linear function and forms the upper (outer) bound for the entire possible convex limit surfaces on the π -plane.

In general, one-parameter criteria are used for those materials having the same strength both in tension and in compression ($\sigma_c = \sigma_t$). Two-parameter criteria are used for those materials which have the SD effect and hydrostatic stress effect (its tensile strength is lower than its compressive strength, ie, $\sigma_c > \sigma_t$). It is better to use three-parameter criteria for those materials having uniaxial compressive strength not equal to the uniaxial tensile strength σ_t , and the equal-bi-axial compressive strength σ_{bc} not equal to the uniaxial compressive strength σ_c ($\sigma_c \neq \sigma_t \neq \sigma_{bc}$). The multi-parameters criteria are used in more complex cases. One-parameter and two-parameter criteria are special cases of the three-parameter criteria. No single model or criterion, however, has emerged which is fully adequate.

The unified strength theory may be a better criterion, which can be adapted for more kinds of materials. It is able to reflect the fundamental characteristics of materials, viz SD effect (different tensile and compressive strengths), hydrostatic pressure effect, normal stress effect, and the effect of the intermediate principal stress, and give good agreement with existing experimental data. The yield criteria for metals and the unified yield criterion are special cases of the unified strength theory.

The unified strength theory is physically meaningful and can be expressed by a mathematically simple equation to the maximum extent possible. It has a unified mathematical model, and a simple and explicit criterion, which includes all independent stress components; it is linear, ie, it is easy in applications to obtain an analytic solution. It is also easy to use in computational implementation for a numerical solution. The singularity at the corners can be overcome simply.

The unified strength theory is not a single criterion; it is a system, a series of continuously variable criteria covering all the region from its lower bound to its upper bound. Most previous failure criteria and yield criteria are special cases or approximation of the unified strength theory. In other words, they can be deduced from the unified strength theory. Moreover, a series of new criteria, which were not formulated before, can be introduced from the unified strength theory.

The unified strength theory has been generalized to formulate the unified slip line field for plastic plane strain problems [881], the unified characteristic line field for plastic plane stress problems [882,883], and axisymmetric problems [884].

The generalized unified strength theory is also suitable for different types of materials under various stress states, but the minimization of the number of material parameters sufficiently representing material response is also demanded. The unified strength theory incorporates various failure criteria from convex to non-convex. It encompasses well-known failure criteria as special cases or linear approximations, and establishes the relations among various failure criteria.

Strength theories (yield or failure criteria) have been widely used in the strength analysis of structures. In recent years the theory of structures has been undergoing a major change in design philosophy: the transition from elastic analysis to that in which the plastic reserves of the material are utilized. A partial exploitation of the plastic properties of materials was allowed by the standards of many countries for the design of structures. Strength theories are also widely used in the slip line field of plane plastic strain, characteristic line field of plane stress and axial symmetric plasticity problems, linear and non-linear analysis of structures by FEM, BEM, Discontinuous Deformation Analysis (DDA), Numerical Manifold Method (NMM), and others.

Strength theory is now generalized not only to perfect elasto-plastic and hardening problems, but also strain softening, elasto-brittle-plastic behavior, discontinuities, localization and bifurcation, microcrack propagation, viscoplasticity, post-critical response, fatigue, fracture, damage, mesomechanics, soil-water characteristics of unsaturated soils, strain

gradient plasticity (Fleck and Hutchinson *et al* [710,1036,1046]), and other areas. Strength theory was also applied to dynamic yield surfaces (Ziegler *et al* [869,1002,1003,1053,1054]), SPH (Smoothed Particle Hydrodynamics, Libersky and Petschek [1044]), thermomechanics [754], etc. A rheology based on the Mohr-Coulomb yield criterion has been implemented in the framework of SPH. A simulation of broken-ice fields floating on the water surface and moving under the effect of wind forces was obtained by Oger and Savage [1045].

A series of researches were carried out to show the effects of strength theory on the analytical results of load-carrying capacities of structures, eg, Humpheson-Naylor [118], Zienkiewicz-Pande [119], Li-Ishii-Nakzato [185,187], Guowei-Iwasaki-Miyamoto [283], and others. Choosing of yield criteria has a marked effect on the prediction of the Forming Limit Diagram (FLD). This conclusion was given by Chan [962], Wagoner and Knibloe [958], Frieman and Pan [960], Cao-Yao-Karafilis-Boyce [949], and Kuroda and Tvergaard [963]. The effects of failure criteria on deformation and discontinuous bifurcation, localization behavior, etc were researched by Mean-Hutchinson [1153], Tvergaard [252], YK Lee-Ghosh [144], Hopperstad *et al* [959], Zyczkowski [1159], Brunig *et al* [739], Zhang and Yu [1160], and others [1156–1159]. The influence of the failure criterion on the strength prediction of a composite was determined by Dano, Gendron, and Picard [952]. The effects of failure criteria on the dynamic response behavior of structures under moderate impulsive load, on the penetration behavior of high speed impact, and on the analytical results of characteristics field were studied by Ma-Iwasaki-Miyamoto [1052], Zukas *et al* [1051], JC Li, Yu and Gong [535,894], and Yu *et al* [881–884]. The choosing of strength theory has significant influence on these results. The unified yield criterion and the unified strength theory provide us with an effective approach to study these effects [276–287,881–884,893–896,1160].

According to Young [1], strength theory was the title of a paper written by Timoshenko at the beginning of the 20th century, and was further a section of some books [2,3]. Strength theories or yield criteria became a chapter in some courses, such as Mechanics of Materials, Plasticity, etc in the 1950s. Strength theory became a course for graduate students in Xi'an Jiaotong University in 1985, and a course for students in Xi'an Jiaotong University in 1993. Some books regarding strength theory or failure criteria have appeared recently [50,55,56,273]. Two Proceedings relating the strength theory were published [1147,1148].

It is very important to choose a reasonable strength theory (yield criteria, failure criterion, or material model) in research and design. The results of research and designs depend strongly on the choice of strength theory in most cases. The selection of the correct strength theory becomes even more important than the calculations, as indicated by Sturmer, Schulz, and Wittig [797]. The bearing capacity of structures, forming limit of FEM simulations, size of plastic zones, and orientation of shear band and plastic flow localization will be much affected by the choice of strength theory. More experimental results of strength of materials

under the complex stress state, and more accurate choices of strength theory are demanded for research and engineering application in the future.

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